EFFECTIVE PLANNING AND ALLOCATION OF FIRE PREVENTION MANPOWER

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EFFECTIVE PLANNING AND ALLOCATION

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SUMMARY

The purpose of this research was to structure several important decision problems concerning allocation and management of municipal fire prevention manpower, and to develop the necessary models and solution procedures to analyze these problems. The operations of the Atlanta Fire Department were used as the basis for structuring these problems and for formulating their mathematical models. The methodology developed in the research was applied to obtain solutions for these problems for the Atlanta Fire Department.

Three problems were considered in the research. These are (1) how often to conduct scheduled, routine inspections each year in each type of building occupancy in each area of the city, (2) how to divide the city into districts for inspectors, and (3) when to schedule each routine building inspection. These three problems are interrelated, but the approach taken in this research was to model the first two problems together and treat the third separately. The mathematical models formulated for these problems are mixed integer programming problems. A piece-wise linear approximation is used in the objective function of the problem of determining inspection rates and districts.

A heuristic procedure was developed to obtain a solution to this first "Rate-District" problem. The procedure utilizes "knapsack" problem logic and an extension of the political redistricting scheme of Hess and Weaver in obtaining a set of inspection rates and districts. A

solution procedure was not developed for the scheduling problem.

Extensive data-gathering activities were undertaken by the Atlanta Fire Department to obtain the parameter estimates necessary to apply the model and solution procedure to obtain inspection rates and districts for Atlanta. This effort included a controlled experiment designed to provide an objective estimate of the effectiveness of routine building inspections. The measure of effectiveness used is the annual expected number of fires in each type of building occupancy in each area of the city, which is a function of the annual frequency of inspection. The experimental estimates were combined with subjective estimates provided by management to obtain a composite estimate of the "effectiveness function" for inspection frequency.

The results of the experiment do not indicate an increase in effectiveness of higher inspection frequencies over lower frequencies. However, the experiment is to be continued for an additional six months to provide more conclusive data.

A set of inspection frequencies and inspection districts were obtained from the Atlanta data and presented to department management for consideration. The districts found were significantly better than the current districts used by the department in terms of the balance of the assigned annual workload among the districts. However, the average compactness of the proposed districts is slightly greater than the current districts (primarily due to two "single area" districts currently in use). It was recommended that the department continue data gathering activities to improve parameter estimates and resolve the model before adapting new inspection rates or districts.

CHAPTER I

INTRODUCTION

Fire protection is one of the primary community services provided by most local governments. In larger cities, fire protection is provided by a full-time, professional fire department. One of the basic activities carried out by these departments is fire prevention.

The prevention of fires can result from many factors other than fire department efforts. However, as being used here, fire prevention is the set of activities performed by fire department personnel for the purpose of preventing the occurrence of fires and reducing the potential seriousness of fires. Typical fire prevention activities include building inspections and education of public and private groups. These activities are usually the responsibility of a separate division within the department, such as a "Fire Prevention Bureau."

Many cities maintain fairly large fire prevention groups or bureaus. While some of these groups are organized and operate differently, most conduct the basic fire prevention activities of building inspections and prevention education for the public. In doing so, they are faced with similar decisions in planning and carrying out these operations. Such problems include how often to conduct building inspections, which buildings to assign to each available inspector, and how to allocate man-hours between inspection activities and public education activities. These and other similar decisions constitute important and challenging problems. Yet practically no research on their analysis has been reported.

Purpose

The purpose of this research is to define and structure the basic planning problems encountered in conducting fire prevention operations. Formulation of mathematical models of these problems and development of solution procedures are also part of the goals of this research. In addition, the research is intended to lay the foundation (in the form of data collection mechanisms) for future analysis of the productivity of effectiveness of both fire extinguishment and fire prevention services.

Problem

The general problem being addressed in this research is "How can fire prevention resources be most efficiently and effectively used?" This problem involves several specific planning or operational decisions. Four such decisions are:

1. "At what rate (inspections per year) should buildings of each occupancy type be inspected?"

2. "How should the city be divided into territories or districts such that there is one district for each inspector?"

3. "When should each routine inspection be scheduled?"

4. "How should resources be allocated between building inspections and public education?"

These decisions constitute planning problems which are relevant to most municipal fire departments. Proper analysis of these problems will aid a department in using their prevention manpower in the most efficient and effective manner.

Approach

To achieve the goals of this research a project was undertaken with the Atlanta Fire Department. Observations made during the project are the basis for most of the problem structure and model formulation results. Additional inputs from outside sources were also used. These included reports from other fire departments and research organizations such as New York City-Rand and the National Fire Protection Association (NFPA).

Estimates of the effectiveness of alternative inspection rates are necessary to analyze the planning problems. The approach used in this research was to obtain both empirical (or objective) and subjective estimates. A combination of these is used as input data for estimating the parameters in the planning models.

The objective estimates are based on the results of a controlled experiment conducted during the project. Each of the 20 full-time inspectors in Atlanta's Fire Prevention Bureau was involved. A computerized data base was set up to accumulate information on both inspection results and fire occurrence. The results of the experiment were obtained from an analysis of this data.

The subjective estimates used to supplement objective data were provided by management personnel in the Fire Prevention Bureau. Consensus estimates were developed through group discussions in which the author, Assistant Fire Marshall, and two Supervisors participated. Estimates of other parameters in the models were obtained by a variety of means including special studies and historical data analysis. The accuracy of all parameter values will be enhanced as data is collected in future department operations.

The problems under consideration are interrelated in that the solution to one affects the solution to the others. However, simultaneous analysis of these problems is extremely difficult because of size and complexity. The approach used in this research is to treat the "rate" and "district" problems simultaneously, but treat the scheduling problems independently (and sequentially). Heuristic solution procedures are used for problems analyzed in the research as exact procedures appear to be impractical due to the size of existing applications.

Scope

Of the four planning problems, only the first three are discussed in this research. Further, only the first two are completely analyzed using collected data from the Atlanta area.

Another limiting factor on the scope of this research is that not all of the results of the research can be directly applied to other fire departments. The models developed are based on the operations of the Atlanta Fire Department. Other departments with different operating policies or organizational structure may not be able to apply the models without modifications. However, extensive changes are not likely to be required.

Presentation

The first section of the body of this report (Chapter II) concerns the background necessary for understanding the problems under consideration. The routine operations and associated planning problems of both fire extinguishment and fire prevention services are described, and relevant literature is also mentioned.

Chapter III describes the problems in detail. The criteria and constraints involved in each problem are discussed, along with the interrelationships that exist between the problems.

The analysis of these planning problems depends on the measurement of inspection effectiveness. Chapter IV contains a discussion of the definition of a measure of effectiveness and a description of the derivation of objective and subjective estimates of inspection effectiveness.

Chapters V and VI are concerned with the analysis of specific planning problems. Chapter V contains a description of the mathematical model of the problem of determining optimal inspection frequencies and inspection districts. A description of the solution procedure is also given. Chapter VI contains similar topics related to the problem of determining a weekly schedule for inspections.

The application of these models to the specific Atlanta data base is discussed in Chapter VII. A description is given of data-gathering efforts, estimation procedures, solution results, and implementation efforts. Finally, the conclusions of the research are given in Chapter VIII along with a description of potential extensions to the current research.

CHAPTER II

BACKGROUND

As mentioned earlier, part of the purpose of this research is to define and structure the basic planning problems encountered in conducting municipal fire prevention operations. The basis for this problem structure, as well as for the associated models, is the set of operations of the Atlanta Fire Department. A research project was undertaken with the Atlanta Department to structure and analyze these problems as they exist in the Department. This chapter is concerned with a description of the operations of the Department, the environment in which they are conducted, and the research project itself. In addition, literature relevant to the analysis of the planning problems is mentioned.

The Atlanta Fire Service System

The city of Atlanta is an urban area covering 129 square miles with a population of approximately 502,500 people. It is located within a 5 county, 1724 square mile metropolitan region of 1,302,000 people (1969 census). There are areas within the city with a high density of construction, and others with sparse construction. The high density areas contain most of the high-rise structures in the city and most of the high-value districts. These areas constitute a designated "fire zone" within the city. Building standards are higher within the zone due to the increased hazards of fire spread. This environment affects the amount and type of resources needed by the Fire Department as well as Department planning (e.g, location of stations, dispatching rules).

The Atlanta Fire Department consists of approximately 1000 men and 75 fire-fighting apparatus. The department performs four basic activities: extinguishment of fires (and related tasks), prevention of fires, support (of prevention and extinguishment activities), and non-fire related activities (rescue, etc.). Over 900 of the men are involved in extinguishment work, while only 23 are assigned to prevention activities. Extinguishment resources are organized into 6 battalions with a "chief" officer in charge of each battalion. A "Fire Marshal" (also a chief officer) is responsible for fire prevention activities [1]. An organization chart of the fire department is shown in Figure 1.

The two primary activities of the department are fire suppression and fire prevention. The operations involved in these two activities are described below. Also discussed are the major problems that face the department in planning and carrying out these operations.

Fire Extinguishment

Operations. Alarms are received by the Department at an average rate of 1.72 alarms per hour [1]. Alarms are received via telephone, call boxes, police units, and other means. When a call is received some complement of extinguishment resources is dispatched to the location of the alarm. The type and number of vehicles dispatched as a response force depends on (1) the suspected nature of the emergency and (2) the location of the emergency. For example, if an alarm is received by telephone and the caller indicates a trashbin is on fire, a response force of

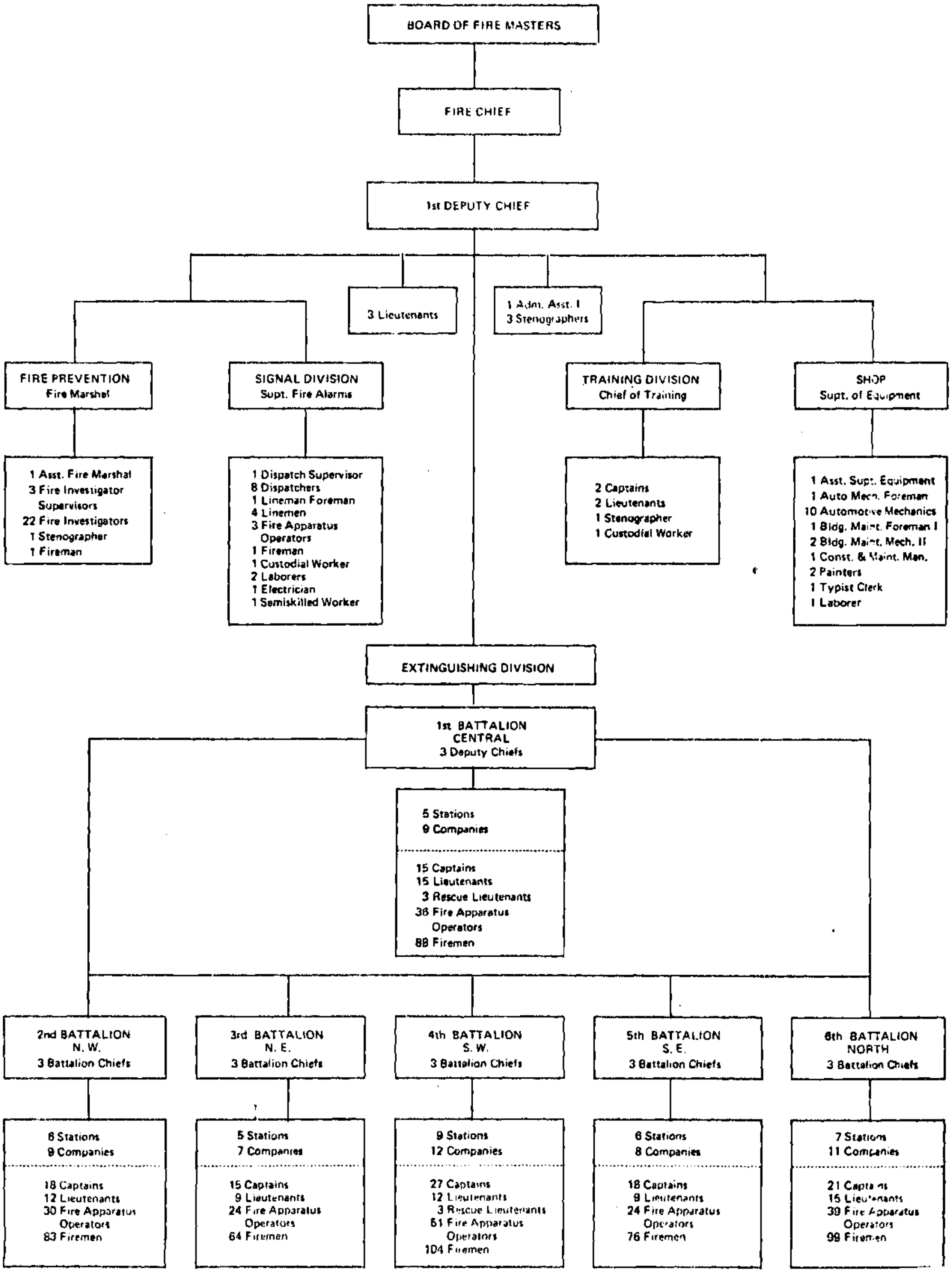

Figure 1. Organization Chart for the Atlanta Fire Department

one engine company would normally be dispatched. However, if the alarm is received from a call box (without voice communication) located in the downtown area, a force of four engine companies, two ladder companies, and a chief officer would be sent. Normal dispatching policy is to send two engines, one ladder, and a chief officer to calls in "low-value" areas, four engines, two ladders,and a chief officer to calls in "high-value" areas (inside the fire zone), and one engine to nonstructure fires.

The specific companies to send to a call are determined by a central dispatcher. His decision is based on predetermined assignments. Each block in the city has been assigned to the "nearest" response companies (two engines and one ladder, or four engines and two ladders). "First alarm" as well as "help call" assignments for specific street addresses have been microfilmed and are available to the dispatch via an automatic retrieval machine. These microfilmed assignment records are similar to "dispatch cards" used in other cities [2].

At the scene of the fire all units (engine or pumper companies, ladder companies, and auxiliary units) are under the command of the ranking officer on the scene. This is usually the responding battalion chief. He is responsible for all operations on the "fireground" (or "fire scene") and for placing "help calls" as he determines additional men and (or) equipment are needed.

Operations at the fire scene may include forcible entry into buildings, ventilation of building, manning fire hoses and operating engine pumps, rescuing and evacuating civilians, and salvage. The need for each of these activities depends primarily on what's burning and on the extent of the fire upon arrival. The commanding officer assesses these and other factors and makes a decision as to how to fight the fire. Specifically, he determines which activities are to be undertaken, when, and to what extent.

The effectiveness of the activities undertaken at a particular fire depends on four basic factors: the equipment used, the number of men used (and their level of proficiency), the time the activity is begun, and the extent of other activities also being performed.* For example, consider a fire in a one-story, single family dwelling. The effect (in extinguishing the fire) of one "booster hose" applied only three minutes after the fire began might be greater than four 1½-inch hoses applied 15 minutes after ignition. Or, six 1½-inch hoses applied 30 minutes after ignition might be more effective than two 2½-inch hoses applied at the same time (however, there must be enough men available to handle the six 1½-inch hoses, or the 2½-inch hoses).

The level of these four factors present at a fire appears to depend on both controllable and uncontrollable influences. Controllable influences include the dispatch decision of how many of each type apparatus to send on the first alarm, the decision of how many men are to ride each apparatus, the decision of how many men to handle each hose, and the decision of where to locate or station each apparatus (this affects response time, and therefore time of use of equipment at a fire).

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*This and similar observations made in this chapter are based on preliminary sampling of field data and on subjective observations from Department personnel.

Uncontrollable influences include time of fire ignition, condition of the structure burning, congestion of people and structures at the fire scene, and location of the fire in the structure.

Planning. Several problems arise in planning the operations of fire extinguishment activities. Until recently, solutions to these problems resulted primarily from guesswork or chance. However, much research has been conducted in the last five to six years in an attempt to quantify these problems and determine rational solutions.

A basic problem which has received most of the attention of recent researchers is where to locate fire stations. Hogg [9], Colner and Gilsinn [10], Chaiken and Larsen [11], Carter and Ignall [12], Raouf [13], Santone and Berlin [14], Larson and Stevenson [15], and others have reported efforts in analyzing this location problem. A closely related problem which has been treated independently is the "relocation problem." The decision concerns which companies to "move up" to vacant stations when "working" fires are in progress, and which stations to move them into. The only published research on this problem known to the author is by Sversey [16], and Kolesar and Walker [17].

Another planning problem concerning extinguishment resources is the determination of response districts. That is, the determination of locations in the city to which a given company is to respond on a first alarm call. Second and higher alarm districts must also be set for each company. The approach normally used is to construct these districts so as to minimize city-wide response time. Chaiken and Larson [11], Carter *et al.* [18], Keeney [19], Larson and Stevenson [15], and others have reported research on this problem.

There are other planning problems which have received little or no attention. For example, the determination of decision rules for the number and type of units to dispatch to an alarm; the desirability of "flying squads" (manpower transport vehicles only); the use of patrolling rather than stationary fire companies; and the determination of the best manning levels for each fire company, and all questions that have not been adequately analyzed.

The analysis of these and other such planning problems must have as input estimates of the effectiveness of alternative solutions to these problems. For example, what is the effect on the expected dollar damage of fires in the city when dispatching four engine companies to all fires rather than two companies? Unfortunately, little research has been reported on defining and (or) measuring effectiveness of alternative strategies for fire-scene activities. Fisher and Midler developed theoretical and numerical "fire-suppression" functions relating several extinguishment activities to an indicator of structural damage [3]. Their empirical results are based on a regression analysis of 134 fires occurring in the Chicago area in 1968. Some of the factors used as independent variables were maximum rate of water application, total manpower used, "pre-burn" time, man-hours of ventilation, forcible entry, extinguishment, and salvage. Their analysis indicates that extinguishment man-hours (hose work) is the most effective structural damage reducer.

In 1969, the Dallas Fire Department conducted simulated tests of fire-scene activities in order to study the effects of alternative apparatus manning policies [4]. Times to perform various activities (rescue, hook-up and reach specific hose-work stations, etc.) were recorded for manning policies of three, four, five, and six men per engine and ladder company. Their results indicate a significant relationship between performance time and manning level. However, the question remains as to what effect such decreases in time has on fire damage.

Unlike the literature on empirical studies of effectiveness of fire extinguishment activities and operations, much research has been reported on the effectiveness of specific fire-fighting equipment and other technology issues. See, for example, Blum [5], Moran [6], Zimmerman [7], and [8].

Fire Prevention

Operations. The second primary activity of the Fire Department is fire prevention. Prevention operations are carried out by the Fire Prevention Bureau. The purpose of these prevention activities is the same as that of extinguishment activities--to reduce human and property damage due to fires. Extinguishment operations attempt to further this objective by waiting until a fire occurs and attempting to suppress it. That is, extinguishment operations are of a contingency nature. Prevention operations attempt to reduce damage by preventing fires from ever occurring, and by reducing the hazards of potential fires (such as keeping theatre exit doors unblocked).

The Atlanta Fire Department has traditionally relied on extinguishment activities to achieve its objective of reducing fire damage.

The major portion of budget expenditures each year goes to resources used in extinguishment operations [1]. However, the author knows of no published empirical evidence that extinguishment operations are more cost-effective than prevention resources in Atlanta or elsewhere.

There are two basic fire prevention activities performed by the Department. First, inspections are conducted of buildings having commercial type occupants (private dwellings are legally restricted from being inspected). Second, fire prevention lectures and education are provided for private and public groups. Most of the available prevention man-hours are spent on inspections rather than prevention education.

Inspections are carried out by full-time inspectors who are members of the Fire Prevention Bureau. ("Familiarization" inspections are also performed by fire station personnel but for the purpose of familiarizing the men with potential fire-scenes rather than preventing fires.) Each inspector has a minimum of five years' experience in extinguishment activities, but receives no formal training in fire prevention. However, on-the-job training is provided each new man in the form of an "apprentice" program.

Each inspector is assigned a territory, or a portion of the city. He is responsible for all prevention activities in that territory (as well as certain fire investigation activities). These activities include (1) scheduled, routine building inspections, (2) nonscheduled, routine building inspections, (3) reinspections, (4) fire report investigation, and (5) indirect work. Each of these will now be briefly described.

"Indirect work" includes activities which are not related to inspections or to fire investigations. This includes fire prevention lectures and special assignments such as photo work. "Fire report investigation" involves the investigation of significant fires (nonzero damage) to determine dollar loss, cause of fire, and extent of human injury. The inspector ofen works with insurance adjustors and with arson investigators (who are also members of the Bureau) in his investigation.

The "demand" for indirect work by an inspector is stochastic and difficult to predict. The demand for fire report investigation by an inspector is also stochastic but is easier to predict because there is a direct relation with the frequency of fires. The time spent on this activity by an inspector is a function of this demand and the time required to make each investigation.

"Reinspections" are follow-up visits to a business which has recently been subject to a "routine inspection." The usual purpose of the reinspection is to check to see if the occupant has corrected previously noted deficiencies. Reinspections are usually scheduled by the inspector (allowing an occupant a specific amount of time to comply with violations notices) but may be called by the occupant. The demand for reinspections is related to the number of routine inspections conducted (which is controllable) and to the number and extent of violations found in each inspection (which is a random factor). The time spent in reinspection activity is a random variable, dependent on the required number of reinspections and the time to complete each reinspection.

"Scheduled, routine inspections" are building inspections which are scheduled at the beginning of a planning period. The inspection consists of a walk-through by the inspector (usually accompanied by a representative of the occupant) in which hazards having potential to cause a fire or to cause serious damage in a fire are noted. The inspector uses a standard code (nationally recommended by NFPA) as a guideline in identifying hazards and citing violations. This code has recently been written into law as a city ordinance in Atlanta. Unlike a "reinspection," a routine inspection is not aimed at an already known hazard and time must be spent in all areas of the building.

The Bureau has complete control over when and where scheduled, routine inspections are to take place. This allows weekly assignments to be given to each inspector of places he is to inspect. The assignments result from two decisions: the frequency of inspection of each type occupancy (schools, hotels, etc.), and the scheduling of when these inspections are to take place.

In the past, determination of inspection frequencies has basically followed NFPA guidelines. "Institutional" occupancies (school, nursing homes, etc.) have been inspected at least four times a year, and structures requiring fire department permits have received priority over non-permit structures. No formal guidelines are available for occupancy other than institutional.

Scheduling of inspections is done in an implicit manner. Each address in the city which is subject to inspection is listed on a "route slip." These slips are organized in a file according to occupancy type (the Bureau uses 66 primary classifications of occupancy) and inspection territory. Each inspector is given a batch of route slips for his territory. He inspects the addresses on these slips as he can. When he finishes these inspections, he is given another batch to work on. A supervisor (there are two in the Bureau) selects which route slips to give the inspectors. This is done by beginning with the first occupancy type listed in the permit section of the route slip file and selecting an arbitrary number of slips. Additional selections of batches of route slip assignments are made by progressing through the permit section in this manner.

The permit section for each territory is "reworked" several times before nonpermit occupancies are ever inspected. The exception to this procedure is that nonpermit institutional occupancies are inspected at four predetermined times during the year. As a result of this procedure of assigning inspections, the schedule of when to inspect each address is currently somewhat arbitrary.

"Nonscheduled, routine inspections" are routine building inspections which are not planned or scheduled at the beginning of a planning period. The need for such an inspection arises due to changes in the occupancy type of a structure, drastic change in ownership of a business, or creation of a new business in a vacant or new structure. For example, 211 Tech Avenue might have been a restaurant at the beginning of the year. Under the current scheduling procedures of the Bureau, this restaurant would be inspected once this year. However, this address would be inspected a second time this year if the restaurant left and a barber shop moved in.

The demand for nonscheduled, routine inspections is random and is not a function of basic routine inspections. The time required to make such an unscheduled inspection is also random, but is the same as the time to conduct a scheduled (routine) inspection in the same building. This time appears to be a function of (1) the occupancy type, (2) the size of the structure, (3) the layout of the structure, (4) the number of deficiencies present, and (5) the attitude of the occupants [20].

Planning. There are four basic problems in planning the operations of fire prevention resources (man-hours). These are (1) determination of the frequency of inspection for each occupancy type in each area, (2) determination of territories for inspectors, (3) determination of a schedule of which addresses to inspect and when, and (4) the allocation of man-hours (and monies) between inspections and fire prevention education.

To the author's knowledge no research has been reported on any of these problems. As in the case of extinguishment planning problems, proper analysis of these questions must rely on knowledge of the effectiveness of alternative solutions. The ultimate measure of effectiveness must be expected damage (human and property) of fires. As stated earlier, prevention activities (inspections and education) are intended to affect this measure by reducing the chances of a fire occurring and by reducing the potential hazards in fires that do occur.

The current research is focused on these planning problems for prevention resources. Detailed discussion of the problems is presented in Chapter III.

Relevant Literature

A considerable amount of literature has appeared in recent years concerning policy issues for fire suppression operations, but none has been reported on policy issues for fire prevention operations. However, research has been reported on the *methodology* for analyzing problems similar to the fire prevention planning problems under investigation.

The determination of inspection districts is similar to the problem of determining legislative districts in a county or state. This so-called "political redistricting" problem has been considered by Hess and Weaver [23], Balinski [24], Garfinkel [22], and others. The relationship between the inspection districting problem and the political redistricting problem is discussed in Chapter V.

A general formulation of the model of the political redistricting problem is as follows [21]:

Let N = the number of districts to be determined.

I = the number of areas to be composed into districts.

p_j = the population of the j*th* area.

d_{ij} = the distance from the center of area i to the center of area j.

Δ = the allowable percentage deviation of a district's population from the average.

x_{ij} = 1 if area j is assigned to a district centered at i; 0, otherwise.

Then the model is

$$\underset{\underline{x}}{\text{minimize}} \sum_{i=1}^{I} \sum_{j=1}^{I} d_{ij}^2 p_j x_{ij}$$

$$\text{S.T.:}\quad (1)\quad \sum_{i=1}^{I} x_{ij} = 1, \qquad\qquad \text{for all } j,$$

$$(2)\quad \sum_{i=1}^{I} x_{ii} = N,$$

$$(3)\quad (1-\Delta)\bar{p}x_{ii} \le \sum_{j=1}^{I} p_j x_{ij} \le (1+\Delta)\bar{p}x_{ii}, \qquad\qquad \text{for all } i,$$

(4) All districts are contiguous.

There are three criteria reflected in the above model. They are (1) the compactness of each district, (2) equality of population among all districts, and (3) the contiguity of each district. For a discussion of these criteria and how they can be measured, see [21], [22], or [23].

Garfinkel's approach is the only procedure found in the literature which guarantees to produce a global optimum. The other approaches are heuristic in nature. Garfinkel uses a two-step process. First, an exhaustive set of possible districts is developed which are feasible with respect to all three criteria mentioned above. Then, a subset of districts is selected which includes all areas and which minimizes the maximum population deviation.

Unfortunately, the exact approach suggested by Garfinkel and applied by Garfinkel and Nemhauser [25], does not appear to be practical for large problems. They report being unable to solve a problem with just 55 areas [25]. The difficulty apparently arose from the fact that using tight limits on the criteria of Phase II (their second solution step) with looser Phase I limits (their first step) led to too many variables as candidates in Phase II [21].

The method used in this research to solve the inspection districting problem (having 118 areas) is based on the Hess and Weaver approach. A discussion of their procedure and the modifications made is given in Chapter V.

The analysis of the problem of determining inspection rates utilizes methodology also used to solve "knapsack" problems. Dantzig [26] gives a good discussion of the all-integer version of this problem and its solution characteristics. Woolsey [27] also discusses this problem but in the framework of a capital budgeting decision. There are numerous other articles dealing with the knapsack problem but no attempt will be made here to list or describe them.

The Research Project

The work presented in this report is the result of a research project conducted with the Atlanta Fire Department. The project was begun in December, 1972, with most of the work completed by October, 1973. However, additional data gathering was carried on after October.

Purpose

The primary objective of the project was to provide the background information and data necessary to analyze the fire prevention planning problems as they exist in Atlanta. As a "spin-off" from the project, it is anticipated that a permanent computerized information system will be installed and used by the Department. Such a system will allow the data-base used to analyze prevention planning problems to be enlarged. This will enable future applications of the models developed in the project to be more accurate. Also, the additional data will be useful for future analysis of *extinguishment* planning problems and development of budget justification procedures.

Scope

There was extensive involvement in the project by personnel from all levels and divisions of the Fire Department. Fire station personnel assisted in data gathering and coding. Battalion chief officers also assisted in data gathering. In addition, all chiefs made on-the-scene estimates of several "unquantifiable" factors related to the results of a fire and response efforts applied to the fire. The battalion chiefs also played a major role in providing inputs to problem structuring, modelling, and effective data gathering forms and procedures.

Dispatching personnel also were part of the data gathering efforts. They collected information communicated from the fire scene by pump operators and battalion chiefs (or their "aids" or assistants). Most of the efforts of station and dispatching personnel, and of batallion chiefs were concerned with extinguishment operations.

All members of the Fire Prevention Bureau were heavily involved in the project. In fact, each of the 20 inspectors spent the majority of his time during the period April, 1973, through September, 1973, in data gathering (as part of the experiment described in Chapter IV). Management and supervisory personnel in the Bureau provided data inputs (for the planning models) as well as the major inputs into the formulation of the prevention planning models. The Records Division (part of the Bureau) was responsible for consolidating various data forms, checking their completeness, and coding them.

Other agencies in the city were also involved in the project. Limited input as to the usefulness and value of the results of the project was provided by "Budget Analysts" in the Finance Department (as well as by budget analysts in DeKalb County, Georgia). The Planning and Zoning Department provided data on buildings (age, occupancy type, etc.) from their "Plan File." The Systems Group (in Data Processing) provided input into development of computer programs, and became familiar with the research problems and solution approaches. It is intended that this group will provide continued "technical assistance" to the Fire Department in the maintenance of the planning models and analytical computer programs.

Activities

Three types of activities took place during the project: modelling, data collection, and data analysis. Each of these activities is briefly discussed below.

Modelling. Planning models were developed for three operating decisions for fire prevention manpower. These models and the specific problems they are concerned with are discussed in Chapters III, V, and VI. The formulation of these models was based on interpretation of the underlying problems, the department's objectives, operating policies, etc. Input for this came from direct observation by the author of department operations and from discussions with management and field personnel.

Data Collection. Three basic types of information were collected. First, "fire incident" data was collected. For every building fire or other emergency in which people were injured, data was gathered on the response effort (equipment and men dispatched, response time, etc.), extinguishment activities (number and type hoses used, time of hose use, etc.), and actual and potential fire results (cause, time of origin, damage, etc.).

Fire incident information was collected on three separate source documents and then coded on a consolidated form. The three documents were the normal "Fire Report" used in daily operations of the department, a special form compiled by the dispatcher, and a special supplementary fire report compiled by the battalion chiefs. Copies of these forms as well as of the consolidated coding form are included in Appendix I.

The second type of information collected was the results of building inspections. Each time an inspector completed a building inspection he filled out a special coding form indicating the date of inspection, the results, etc. This form is included in Appendix I. The inspections conducted and coded during April-September were part of the controlled experiment to determine the effectiveness of alternative inspection frequencies. This experiment is described in Chapter IV.

The third type of data gathered concerned information needed in the planning models. This includes the number of each type of occupancy in each census tract, the standard time to conduct an inspection, estimates of the number of fire reports to be investigated in each census tract, etc. This information was collected through special projects and from historical records. The specific information required will be discussed in Chapter VII.

Data Analysis. The third basic activity of the project was analyzing the data collected as described above. This included analyzing the controlled experiment to determine the effectiveness of inspections as a function of inspection frequency. The record of fire incidents was compared to the record of inspections to establish the results of the experiment (that is, the probability of fires in each class of occupancy types, given each experimental inspection rate). Two years of historical data were also used to supplement these results. The experiment and its statistical analysis are described in Chapter IV.

Another type of analysis performed was the solution of the planning models. The algorithm to solve the rate determination and districting problems described in Chapter V were coded, checked with example problems, and used to analyze these problems using the actual data collected. The results were examined and presented to the Department Management. These efforts are described in Chapter VII.

CHAPTER III

PLANNING PROBLEMS

In Chapter II several problems that arise in planning the operations of fire prevention manpower were mentioned. The current research considers three of those problems: (1) determination of inspection rates, (2) determination of inspection districts, and (3) determination of a schedule for inspections. These problems are described in detail in this chapter. The discussion includes the criteria, constraints and assumptions involved in each problem. The interrelationships among the problems and the approach used to deal with these are also described. Model formulations are presented in Chapters V and VI.

Determination of Inspection Rates

Each year the Fire Prevention Bureau must decide how many times to conduct scheduled, routine inspections in each building in the city. Because the hazards (to life and property) in all buildings of the same type occupancy are similar, the decision on the yearly inspection rate is made for each *occupancy type* rather than for individual buildings. However, there are specific buildings that have unusual hazards which are treated separately. The decisions on these inspection frequencies are left to the inspector responsible for those buildings.

It may be desirable to set different frequencies on the same occupancy type in different parts of the city. For example, the potential seriousness of fires in concentrated, densely populated areas is greater than in sparsely populated areas where "conflagration" (fire spread) is not a problem. Atlanta's "fire zones" contain most of the potential conflagration areas in the city. Fire Prevention Bureau management felt that buildings within the fire zone should be inspected at a greater rate than the same type buildings outside the zone.

Therefore, the decision problem is what should be the yearly frequency of scheduled, routine inspections for each type occupancy in each of the two areas (in the fire zone, and outside the zone). This decision constitutes a resource allocation problem in which the total available inspector man-hours are allocated among all inspection categories. Increasing the inspection frequency of a particular occupancy type in a particular area increases the number of inspections that must be conducted in those buildings. These additional inspections consume man-hours that perhaps should be used to increase the inspection frequency in some other category of inspections. Therefore, all inspection categories (i.e., specific types of occupancy in specific areas) are in direct competition for the available inspector man-hours.

Criteria

The general criterion upon which the inspection frequency decision is made is the total "value" of inspecting all inspection categories at specific rates. Value could be defined in several different ways. Whatever definition is used should reflect the ultimate goal of the Fire Prevention Bureau and the Fire Department. As stated earlier, this goal is to reduce human and property loss by preventing fires and reducing hazards in potential fires. In other words, the goal of prevention operations is to reduce the expected seriousness of fires (expectation is with respect to both the number of fires and the severity of each fire).

However, in planning their operatins, the Bureau uses *potential* seriousness or hazards of a fire rather than the average or *expected actual* seriousness as part of the criterion. For example, even though the average human and property loss in a hospital fire may be relatively small, the *potential* loss of life in a major fire in a hospital is very large. This causes the Bureau to place a high value on preventing fires in hospitals.

Therefore, the criterion to be used for planning purposes is the total expected potential seriousness of building fires (i.e., the potential seriousness multiplied by the expected number of fires). This is the criterion which is used in the current research to evaluate alternative frequencies for scheduled, routine inspections in each inspection category.

Constraints

There are four restrictions on the inspection rate that can be assigned to each inspection category.

1. *Workload Feasibility*. The available inspector man-hours limits the total number of inspections which can be made (and therefore assigned). The total inspector man-hours available in a year are consumed by all five prevention activities mentioned in Chapter II. These are scheduled,routine inspections; nonscheduled, routine inspections;

reinspections; fire report investigations; and indirect work.

It is assumed, for analysis purposes, that changes in inspection frequency affect only the man-hour requirements of scheduled, routine inspections. The time consumed by the other four operations is considered to be independent of the decision of how often to inspect each occupancy type. Therefore, yearly frequencies of scheduled, routine inspections must be set such that the total man-hours consumed by these inspections plus the man-hours consumed by the other four operations is less than or equal to the total man-hours available in the year.

2. *Frequency Limits.* The second restriction concerns the magnitude of the frequency of scheduled, routine inspections. Because of the relative potential seriousness of certain occupancies, the Bureau sets a minimum limit on the number of scheduled, routine inspections that must be conducted in these occupancies. These limits currently follow NFPA recommendations (these recommendations are given only for "institutional" occupancies).

There are also reasons to limit the *maximum* number of inspections conducted in a building when an inspection takes places some measure of disruption of the occupant's normal activities takes place. Usually, a representative of the occupant accompanies the inspector in his examination of the building. In certain types of occupancies, such as small manufacturing, this disruption can cause severe problems. A negative attitude on the part of the occupant can even be created. If the occupant resents the presence of the inspector, the occupant is less likely to cooperate or to listen to the advice of the inspector. As a result, the inspection may have little or no effect.

In setting upper and lower limits on inspection frequency, Bureau Management preferred that all buildings of the same type occupancy are assigned the same limits. Management saw no reason for distinguishing between individual buildings of the same occupancy type, or between the same occupancy type in different areas. Therefore, limits are set on the frequency of inspection of each occupancy type, regardless of the area in which those occupants are located.

3. *Equivalent Categories.* The third restriction on frequencies is that inspection categories having the same "value" and the same upper and lower frequency limits must have the same frequency. The purpose of this restriction is to ensure that "equivalent" inspection categories have equivalent frequencies. If this constraint were not added, an optimal solution to the "Rate Determination" problem could be obtained in which equivalent inspection categories are assigned different frequencies. This can happen if the solution procedure uses an arbitrary "tie-breaker" rule for choosing entering variables (as in the case of the Simplex Method). Bureau Management feels that available resources should be allocated equally to inspection categories that are equally desirable.

"Equivalent" categories is used here to imply that the probability of realizing the potential seriousness of a fire in those categories is the same. This definition is based on the potential seriousness of a fire, and the probability of at least one fire occurring. It is independent of the total number of structures of the type in question in the particular area.

4. *Integrity Requirements.* The frequencies being determined are the number of inspections *per year*. However, these annual inspection rates will be applied to scheduling inspections over a planning horizon of P years. The annual rates must be set so that they can be spread over these P years. This requires that the rates be integer multiples of the fraction 1/P.

For example, suppose a planning horizon of three years is being used. Then acceptable levels of the inspection frequencies would be 1/3, 2/3, 3/3 (or 1), 4/3, etc. Unacceptable levels would be 1/2, 3/4, 5/2, etc. The interpretation of the fractional level 1/3, say, is one inspection every three years. Therefore, inspection frequencies are constrainted to be integers, or integer multiples of 1/P.

Determination of Inspection Districts

The responsibility for conducting an inspection is assigned to a specific inspector. This is normally done by requiring each inspector to conduct all fire prevention operations in a specific territory or district. The decision of what portion of the city to assign to each inspector as his district is a second planning problem of importance to Bureau Management.

Districts are formed by combining separate areas or regions of the city into mutually exclusive sets or groups. In the past, the Bureau used "fire station inspection territories" as the basic areas to be combined into districts. There are 30 such station territories in the city of Atlanta. See Figure 2 for a map of these territories. However, areas smaller than station territories are more desirable as

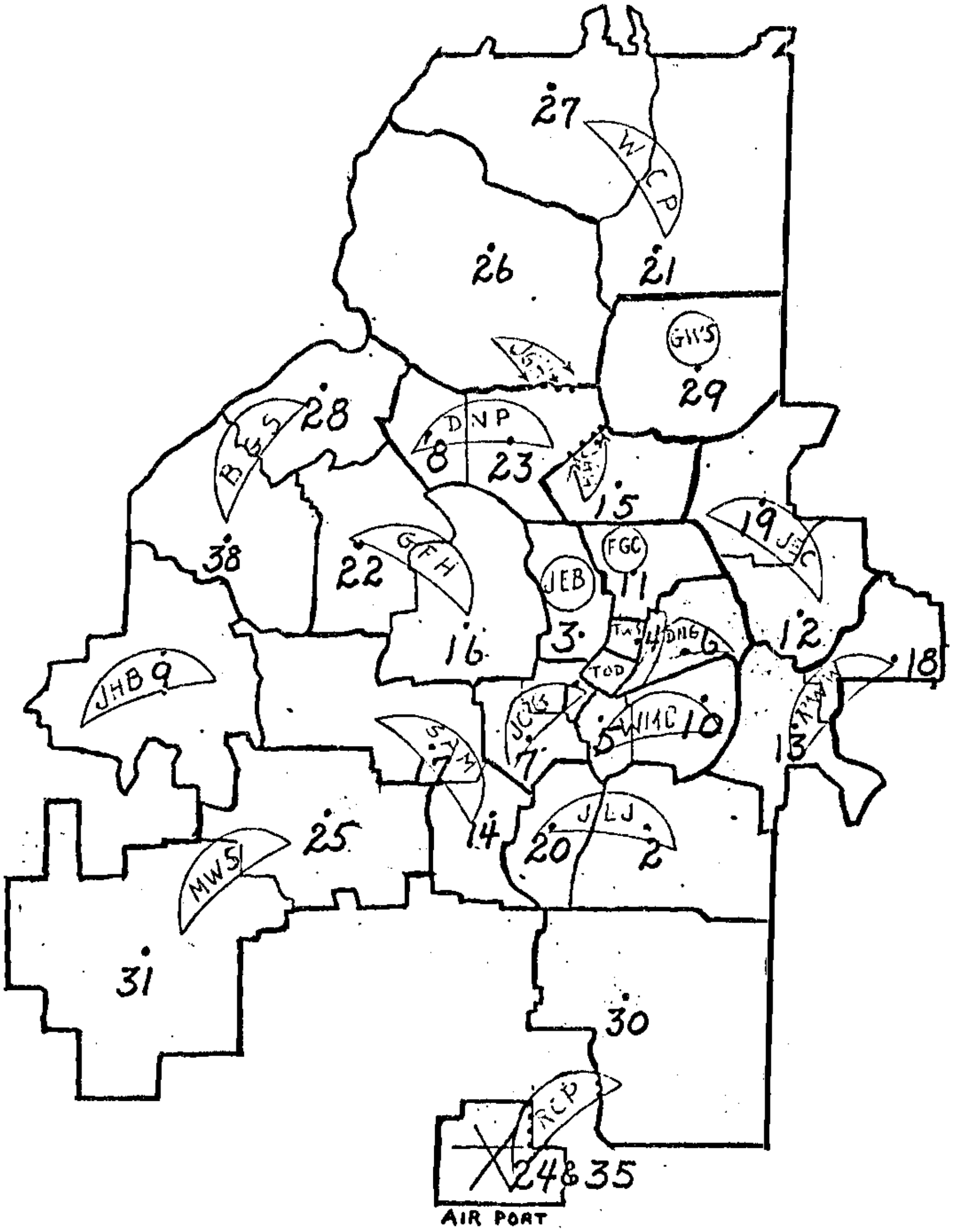

Figure 2. Current Inspection Territories in the City of Atlanta

the basic building blocks of a district. Smaller areas allow districts to have more refined and flexible definitions. However, the smaller the area used as a building block, the larger the data gathering problem.

The approach used in this research was to utilize population census tracts as the basic area building blocks. The city of Atlanta is divided into 118 tracts [28]. The use of census tracts should lead to "better" districts than can be formed with station territories.

Criteria

There are several criteria which appear to be relevant in the Bureau's evaluation of the desirability of a set of districts. The following three characteristics of a set of districts appear to be most important.

1. *Workload Equality*. Each district in a set of districts has a workload associated with it. This workload is the combined time (man-hours) consumed in conducting the five fire prevention operations in the district. The time required by an operation depends on the "demand" for that activity in a district as pointed out in Chapter II, the "demand" for scheduled, routine inspections in an area is deterministic and controllable. In fact, the level of these routine inspections is set by the solution to the "Inspection Rate" planning problem.

The "demand" for the other four operations is random and assumed to be uncontrollable (see Chapter II). The estimated mean value of the demand for each of the four operations is used to compute the level of that activity in a district. The workload requirement for each operation is the level of that operation (e.g., number of inspections) multiplied by the expected time to conduct each event involved in the operation (e.g., an inspection).

It is desirable to have approximately the same workload in all districts. This allows each inspector to have the same level of work. An inspector with several large manufacturing occupancies in his district requiring a large amount of time and effort to inspect (or reinspect, etc.) would have the same time commitment as all other inspectors. Or, an inspector having a district with a large number of fire reports requiring investigation would have the same total time commitment as an inspector whose district was expected to have few fire reports.

Another reason that districts with "balanced," or approximately equal, workloads are desirable is that this allows *equal* service to be given to all occupants. That is, if a district has an excessive workload it will be impossible for all buildings in that district to be inspected at the desired rate (i.e., the rate set in the "Rate Determination" planning problem). This would cause some buildings to be "underinspected."

In a district with a very small workload, the inspector will avoid idle time by conducting more routine inspections than were planned. This causes some buildings to be "overinspected." The same occupancy type which is overinspected in one part of the city may be underinspected in another area of the city, even though the value of inspecting each may be the same. The result is "unequal" service to different parts of the city.

There are several alternative ways to measure workload balance in a set of districts (see [21] for measures used in "political redistricting" problems). The approach used here is to let the workload deviation for a district be the difference between the actual workload of that district and the average workload over all districts. This deviation is examined for each district. The deviation is acceptable if the workload of the district is within a specified per cent of the average workload. This approach is consistent with several models of the "political redistricting" problem (see [21], [22], [23]).

2. *Compactness.* Compactness is an indication of the shape of a district. A compact district is one which is not excessively irregular in shape (e.g., elongated, or S-shaped). There are several ways to measure compactness. Some measures use distances only, some use workload only, and others use a combination of these two [21]. The measure used in this research is the sum of the square of the weighted euclidean distance between the center of gravity of each census tract assigned to the district and the center of gravity of the district itself. The distance is weighted by the annual workload associated with the census tract. The centers of gravity are with respect to the distribution of workload in an area. See Hess and Weaver [23] for a discussion of this approach to measuring compactness, and [29] for calculation of centroid values.

A compact district is desirable for inspection purposes since the travel distances between inspections tend to be smaller than in less compact districts. This would be especially true if inspections were scheduled randomly over the district. However, there is presently no theoretical or empirical relationship between intrainspection distances and compactness known to the author. Therefore, this relation is ignored in the planning problem models (see Chapter V). Nevertheless, Fire Prevention Bureau Management feels compactness is a desirable characteristic.

3. *Contiguity.* Contiguity implies that a person can travel between any two points in a district without leaving the district [21]. In other words, each census tract assigned to a district must adjoin or touch another census tract assigned to the district (unless the district is composed of only one tract). There apparently is no concept of "best" contiguity or measures of contiguity [21]. Therefore, if a district is contiguous, it is acceptable. If it is not contiguous, it is not acceptable.

The primary reason that contiguity is a desirable characteristic of inspection districts concerns the administration of inspection operations. It is not difficult to envision confusion arising in record keeping and in determining "who's responsible for what" if districts are not contiguous and are scattered all over the city.

There are other criteria which may be important to other fire departments. For example, it may be desirable to construct districts in such a way as to maintain certain political or natural boundaries. Or, constructing districts that are homogeneous with respect to specific occupancy types might be important in some departments. However, the three criteria presented here are the only ones being used in the current application.

Constraints

There are two primary constraints that restrict the formation of a set of districts. They are that (1) each census tract must be assigned to one and only one district, and (2) a district must be created for each inspector.

Determination of a Schedule for Inspections

A third planning problem confronting Bureau Management is the determination of *when* to conduct each scheduled, routine inspection planned for each building in the city. That is, once the annual frequency of these routine inspections has been set, the decision must be made as to the specific time interval (day, week, etc.) in which each inspection is to take place. For example, if it has been decided to inspect schools outside the fire zone four times a year, a decision must be made as to when each of these four inspections is to take place in *each* of the schools.

A specific time interval used in scheduling inspections must be defined. Intervals such as an hour, a day, a week, or a month could be used. A small time interval allows more precise schedules to be constructed. But smaller time intervals also increase the size of the scheduling problem (see Chapter VI).

There is apparently little need to have very precise schedules. In fact, it is desirable to give an inspector a large degree of freedom over the scheduling decision for buildings in his district. There are two basic reasons for this. First, it is more practical to allow the inspector himself to schedule these routine inspections around the other four operations for which he is responsible. The "demand" for these other operations is random so that scheduling them far in advance--at the same time scheduled, routine inspections were planned--would be very difficult (see [30], p. 141, for a discussion of similar "dynamic" scheduling problems). However, specifying scheduled, routine inspections to take place in a relatively long interval (a week, say) allows an inspector to conduct other activities, such as fire report investigations, and still meet the preplanned schedule for routine inspections.

A second reason to give an inspector more freedom in the scheduling decision is to enhance the "quality" of inspections. If an inspector has some scheduling freedom, he does not have to be concerned about meeting precise time tables for completing inspections. Consequently, he is more likely to take additional time to talk with occupants about fire hazards, and examine the building more carefully. The result is likely to be a more effective inspection and better relationships with the occupant.

In the current research, a week is being used as the basic scheduling time interval. All scheduled, routine inspections will be planned by specifying the week in which they are to take place. It was felt that this length of time strikes a balance between the opposing goals of having a precise and a flexible schedule.

Another consideration in structuring the scheduling problem is the length of the planning period for which the schedule is to be made. This planning horizon must be long enough to allow scheduling of each of the inspections indicated in the designated inspection frequencies. For example, if a particular inspection category (an occupancy type in a specific area) has an annual frequency of two, then a year is a sufficient planning horizon. However, if a category has a frequency of one inspection per three years (or an annual frequency of 1/3), a one year horizon would not be sufficient to schedule the one inspection indicated by this frequency. In this case, a planning horizon of three years would have to be used.

In the Atlanta application, the smallest lower limit on inspection frequency is 1/2 (one inspection per two years). Therefore, a planning horizon of two years is sufficient for the scheduling problem.

Both the problem of determining inspection rates and the problem of determining inspection districts are "city-wide" problems. That is, in both cases decision variables from all areas of the city are treated in the same problem. However, the scheduling of inspections can be done district by district. In other words, once districts are defined the decision of how to schedule inspections in a district is independent of the same decision in other districts.* Therefore, there is a separate scheduling problem for each district.

Criteria

It would be desirable to use the ultimate goals of the Fire Prevention Bureau as the criteria for the scheduling decision. However,

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*The schedule of inspections in buildings on the boundary of a district can affect the desirability of inspecting neighboring buildings in other districts. However, such effects are assumed here to be insignificant.

the relationship between these goals and alternative scheduling decisions is not well known. For this reason, "internal" criteria which are more directly affected by the scheduling decision are more appropriate.

The criterion which is used in this research is the "weekly workload deviation." This is the week-to-week change in the workload assigned to an inspector. For example, a significant (and undesirable) weekly workload deviation for an inspector would occur if he were assigned 65 hours worth of work (from all five prevention activities) one week, no hours the next week, and 55 hours the following week.

The measure of weekly workload deviation for an inspector being used in this research is the sum of the squared deviation of each week's workload from the average weekly workload. In the above example, the average weekly workload is 40 hours, so that the measure of the total weekly workload deviation is 2450.

Management of the Bureau feels that it is desirable to have as small a week-to-week change in an inspector's workload as possible. This may enhance the morale of the inspectors as well as reduce undesirable idle time. This objective is achieved by minimizing the workload deviation measure defined above.

Constraints

A solution to the scheduling problem for an inspection district consists of a list of the weeks in which each building or address in the district is to be inspected. Feasible solutions are restricted to those schedules with the following characteristics.

1. *"Frequency Satisfaction."* The total number of weeks in which

a building is scheduled to be inspected during the two-year planning period must equal the two-year inspection frequency (which is set in solving the "Rate Determination" problem). This restriction must hold for all individual buildings in the district.

2. *"Weekly Workload Feasibility."* The total man-hours assigned to the inspector in each week must not exceed his available working time for the week. The total man-hours assigned in a week includes the time consumed by all routine inspections scheduled for that week, and the time required by the other four fire prevention activities expected to take place in that district during the week. These are the same workload components that constitute the yearly workload for a district which is required in the districting problem described earlier.

3. *"Timing Feasibility."* There are two "dimensions" to the schedule of inspections in a particular building. One is the distribution of the inspections in that building over time. That is, the specific weeks in which inspections take place. The other "dimension" is the proximity of that building to other buildings being inspected during the same week.

There are two aspects to the distribution of inspections over time. The first is the specific date of each inspection in a building. The date of inspections is important (in the judgement of Bureau Management) in certain types of occupancies. For example, it is desirable to inspect schools the month before classes begin and again during the first month of classes. In other occupancy types, such as small retail mercantile, the specific date of inspection is not thought to be critical.

The second aspect is the length of time between inspections in the building. For example, if one inspection takes place in the second week of the two-year planning period and a second inspection takes place in the 20th week, then the "intrainspection" time is 17 weeks. There are several alternative intrainspection time policies or strategies which can be imposed on the schedule of inspections in a building. One strategy (or scheduling rule) is to assign the inspections *randomly* over time. Another is to assign inspections in such a manner that the resulting intrainspection times are *uniform*, or equal. For example, if there are three inspections to be made in a building during the planning period, a uniform strategy would result in the time between the first and second inspections being the same as the time between the second and third inspections.

At present, there is no evidence of the relative superiority of any of these timing strategies. Based on the judgement of Bureau Management, it is desirable to follow a uniform strategy in Atlanta.

All possible schedules of inspections in a building which are feasible with respect to (1) the date of each inspection, and (2) the intrainspection times, can be included in a finite set, $\{X_{i\ell}\}$. There is one such set for each building in the district. The solution to the scheduling problem is constrained to be a member of the intersection of these sets. That is, the schedule of inspections in *each* building must be a member of the feasible set $\{X_{i\ell}\}$.

4. *"Neighbor Inspection Feasibility."* The second "dimension" to a schedule of inspections in a building is the proximity of that

building to other buildings being inspected during the same week. As in the case of intrainspection time strategies, there are several alternative policies concerning the proximity of inspections which can be imposed on the schedule. A random strategy could be followed in which buildings must be randomly selected throughout the district for inspection in a particular week. This type strategy might prevent occupants from "being prepared" for an inspection (that is, from creating an unnormal, acceptable condition in their building).

Another strategy is to require inspections to take place in buildings in nonadjacent areas. This might produce the same effects as the random strategy. A third strategy would be to require that inspections be conducted in buildings in the same or adjacent areas. This strategy would reduce travel time and thereby increase the number of man-hours available for fire prevention operations. Again, no evidence is currently available as to the relative merits of these alternative strategies.

Those schedules of inspections in a district in a particular week which are feasible with respect to the strategy imposed on the proximity of building inspected, constitute a finite set $\{X_t\}$. There is one such set for each week in the planning period. The solution to the scheduling problem is constrained to be a member of the intersection of these sets. That is, the schedule of inspections for week t must be an element of $\{X_t\}$ for all weeks in the planning period.

Interrelationship of Problems

All three of the problems described in this chapter are interrelated. The solution to any one affects or is affected by the solution to the others. For example, inspection frequencies set in the "Rate Determination"(R.D.) problem affect the workload required in each census tract. The workload in the various census tracts affects two of the three criteria of the "District Determination" (D.D.) problem. Specifically, workload balance in districts is changed as the census tract workloads are changed. Also, the compactness of a district changes as the center of gravity (based on workload distribution) of the district changes.

Another interdependency that exists between the R.D. problem and the D.D. problem occurs because changes in districting solutions change the compactness of the resulting districts. Since more compact districts result in smaller travel time, compactness changes affect the time available to conduct scheduled, routine inspections. Changes in available man-hours (the scarce resource in the R.D. problem) can affect the optimal inspection frequencies.

The scheduling problems (S.D.) are affected by the solution of both the R.D. and D. D. problem. Changes in inspection frequencies can affect the weekly workload of an inspection district. This, in turn, can affect both the workload deviation criterion and the weekly workload constraint in the S.D. problems. Changes in the make-up of districts (assignment of census tracts) affect the number and size of scheduling problems.

The solution to the S.D. problems can have an effect on the solution to both the R.D. and D.D. problems. Changes in the schedule of inspections in a district can change the proximity of the buildings that are inspected in the same week. This, in turn, can affect travel time which affects available man-hours. And changes in available man-hours can change the solution to both the R.D. and D.D. problems.

In order to determine exact solutions to these three problems, they must be solved "simultaneously." Otherwise, suboptimal or infeasible solutions to each problem may be obtained. Geoffrion discusses a multi-criteria optimization approach to determine exact solutions to these type problems [31]. However, application of such an approach to the three problems being analyzed here would be extremely difficult because of the large size of the overall problem and the complex interrelationships that would have to be treated explicitly (see the mathematical models in Chapters V and VI for an indication of the size, complexity, and number of integer variables involved).

The approach used here is to analyze the "Rate Determination" problem ("R.D.") and the "District Determination" problem ("D.D.") simultaneously--in the same mathematical model. The scheduling problems are solved separately, but after the R.D. and D.D. problems. There are two basic reasons for using this solution approach. The first concerns the sequential relationship between the solutions of the problems. The inspection frequencies set in the R.D. problem and the districts defined in the D.D. problem together form the basis of the formulation of the S.D. problem. It is therefore "natural" to solve the R.D. and D.D.

problems before solving the S.D. problems.

The second reason is that it is desirable to analyze as many of the interdependent relations as possible in the same model. This reduces the degree of inexactness of the solution. The size of the R.D. and D.D. problems and the complexity of the interdependencies are not so severe that these two problems cannot be analyzed together. There may be alternative ways to analyze these three problems, but the current approach appears logical, and appears to introduce a relatively small degree of inexactness.

CHAPTER IV

DETERMINATION OF INSPECTION EFFECTIVENESS

The criterion of the "Rate Determination" problem discussed in Chapter III is the expected potential seriousness or hazard of building fires. Expectation is with respect to the number of fires to occur. Routine inspections can affect both the expected number of fires and the potential hazards of a fire. This chapter is concerned with measuring or estimating this effect so that the criterion of the planning problem can be quantified.

The first part of this chapter is a discussion of the definition of an effectiveness measure and an associated mathematical representation or model. The model is equivalent to a model of the incidence of fire. The next portion of the chapter deals with the estimation of the parameters in this model. These parameters are actually functions, $p(z)$, of the annual rate of scheduled, routine inspections. Both objective and subjective estimates are used to estimate this parameter function at three specific inspection rates. The development of both types of estimates is described, along with the procedure used to combine them into a single value. The last part of the chapter is a discussion of the derivation of an "effectiveness function." This is a convex, piecewise linear approximate to the actual parameter function, $p(z)$. It is based on the combined subjective and objective estimates made at the three inspection rates. The effectiveness function is the primary component of the quantification of the criterion of the "Rate-District" problem.

Effectiveness Measure

In this research, the effect of alternative inspection frequencies on the potential *hazards* of a fire will not be considered. The potential seriousness of fires in each occupancy type is assumed to be independent of the rate of scheduled, routine inspection. Therefore, in order to determine the effect of inspections on the planning problem criterion, an estimate of the effect of inspection frequency on the expected number of fires must be obtained. This effect may be different in different occupancy types, and in different areas of the city. Hence, the relevant measure of effectiveness of inspection frequency is the "expected number of fires in each type occupancy in each area."

The difference in the expected number of fires for a particular inspection category (i.e., a particular occupancy type in a particular area) inspected at different frequencies is equivalent to the expected number of fires prevented due to the change in inspection frequency. This difference is an indication of how effective is the change in inspection frequency.

There are several factors which may influence the effectiveness of inspections besides the frequency. For example, the type of occupancy of the structure being inspected can influence the prevention effect of an inspection. Apartment buildings often offer little or no "common access" area to be inspected and little chance for an inspector to spot potential hazards that may lead to fires (most fires in apartment buildings appear to occur in individual apartments which are not subject to inspection). On the other hand, most hazards present in a manufacturing concern are visible to an inspector which increases the chance of reducing hazards and therefore of preventing fires. Other factors which may influence the effectiveness of an inspection are the attitude of the occupants, the nature of the business conducted (within a specific occupancy type), the surrounding neighborhood, the age of the structure, etc. [20].

In order to account for these uncontrollable influences when measuring inspection effectiveness, "fires" can be cataloged into mutually exclusive groups or types. Then, the effectiveness of inspection frequency in reducing the expected number of fires of each type can be examined. Fires can be classified according to the type structure in which they occur. In turn, structures can be classified according to various criteria. In this research, two factors are used to categorize a structure: its occupancy type, and the area in which it is located. Together, these two factors appear to account for the major uncontrollable influences on inspection effectiveness.

Estimation of Effectiveness

In order to estimate the effect of inspection frequencies on the expected number of fires, a model must be formulated relating the level of fire incidence to inspection rate. The general form of the model must first be developed, then numerical estimates of the parameters in the model must be obtained.

Notation

The following definitions will apply to the developments presented in this chapter.

Subscripts

i = geographical areas or subregions in the city ($i=1,2,\ldots,I$).

k = the final classification of occupancy types ($k=1,2,\ldots,34$).

j = a secondary classification of occupancy types used in the experiment ($j=1,2,\ldots,6$).

ℓ = the number of fires per six months in a building ($\ell=1,2,3$ or $>$).

t = a six-month interval ($t=1$ implies April-September, $t=2$ implies October-March).

Variables and Parameters

$y_{ikt}(z)$ = the total number of fires in type k occupancies in area i in the *tth* six-month interval, when those i-k structures are inspected at an annual rate of z scheduled, routine inspections.

$x_{ik\ell t}(z)$ = the number of type k occupancies in area i which have ℓ fires in the *tth* six-month interval, when those i-k type structures are inspected at an annual rate of z scheduled, routine inspections.

$x^{s}_{j\ell t}(z)$ = the number of type k occupancies in the city in which ℓ fires are counted during the *tth* six-month interval during the experiment, in those j type buildings inspected at rate z during the experiment.

N_{ik} = the number of occupancy type k structures in area i.

$N_j(z)$ = the number of occupancy type j structures in the experimental sample which is inspected at a rate of z.

$p_{ik\ell t}(z)$ = the probability of a type k occupancy in area i having ℓ fires during the *tth* six-month interval, when the frequency of scheduled, routine inspections is z per year.

$p_{j\ell t}(z)$ = the probability of a type j occupancy having ℓ fires during the t*th* six-month interval, when the inspection frequency is z.

p(z) = the general form of $p_{ik\ell t}(z)$ and $p_{j\ell t}(z)$ in which the subscripts are suppressed for convenience.

Also, the symbol "^" will be used to indicate an estimate of an unknown value. Other notation will be introduced in the chapter as necessary.

Model of Fire Incidence

The Poisson process has been used by some researchers as a basic model of fire incidence [38]. Specifically, the number of fires in a particular area, with a fixed set of structures, over a period of time is assumed to follow a Poisson distribution.

An alternative approach is to use multiple, independent states of the process of occurrence of fires rather than a single state, the *total* number fires. That is, the alternative is to construct the model such that several possible levels of fire occurrence are directly represented. In the notation presented earlier, fire occurence can be modelled in this alternative approach as follows:

$$y_{ikt}(z) = \sum_{\ell} \ell x_{ik\ell t}(z)$$

The realization of the x variables results from the realization of a series of 0-1 events. An event is the observation of one of the i-k structures (a type k structure in area i) at the end of period t. A "successful" event (having value 1) occurs when one such structure has exactly ℓ fires during that period. For example, $x_{1212} = 5$ results from the occurrence of a fire in 5 different structures of type 2 in area 1 during six-month period 2. In this case, 5 observations of the total number of i-k structures, N_{ik}, were "successes" (i.e., had exactly $\ell = 1$ fires during t = 2) while $N_{ik}-5$ were "failures."

The imputed sampling process that results in a realization of an x value is 100 per cent sampling of the i-k structures. Assuming that the occurrence of fires in a structure is independent of the occurrence of fires in other structures, the sampling distribution is a binomial distribution so that $x_{ik\ell t}(z)$ is a "binomial random variable." Let the parameter of this distribution be $p_{ik\ell t}(z)$.

Using the binomial model, the expected number of i-k fires in period t is

$$
\begin{aligned}
E\left(y_{ikt}(z)\right) &= E\left(\sum_{\ell} \ell x_{ik\ell t}(z)\right) \\
&= \sum_{\ell} \ell E\left(x_{ik\ell t}(z)\right) \\
&= \sum_{\ell} \ell N_{ik} p_{ik\ell t}(z).
\end{aligned}
$$

A major advantage of defining the intermediate variable $x_{ik\ell t}$ having $\ell = 1,2,3$ states rather than modelling only the total number of fires, x_{ikt}, is that more refined analysis of inspection frequency effectiveness is possible. Alternative inspection rates can be examined as to their impact not just on the *total* number of fires in a group of structures but on occurrence of the first fire that might occur in a structure, the second fire that might occur, the third, etc.

There appear to be some types of structures (for example, hospitals) in which *several* fires tend to occur during a six-month period and others (e.g., stores and dwellings) in which at most one fire occurs [20]. It may well be that occupants of some type structures become more "fire conscious" than occupants of other types after an initial fire. This might indicate that the major impact or effect of inspections in structure types tending to have at most one fire would be derived on the initial inspection rather than subsequent inspections. That is, one inspection per year in this type structure might be just as effective as 12 inspections per year.

On the other hand, structures in which multiple fires tend to occur during a time period may require frequent inspection in order that the inspection program have an effect. This would imply that a higher level of inspection frequency would be more effective (in preventing fires) than lower levels. Knowledge of the relative effect of inspection frequencies in structures tending to have ℓ fires per six-months permits the measure of effectiveness, "expected number of fires," to be more accurately modelled. This leads to more efficient use of fire prevention manpower in solving planning problems.

Estimation of Model Parameters

The problem of estimating the effectiveness of inspection frequency is equivalent to the problem of estimating the "fraction defective" parameter, $p_{ik\ell t}$, as a function of inspection rate. The approach being used in this research is to make both an objective (or sampling) estimate of $p_{ik\ell t}$ and a subjective (or prior) estimate of $p_{ik\ell t}$. The two are then combined to produce one composite estimator.

Three values of the inspection rate, z, are used to develop the function $p_{ik\ell t}(z)$. They are (1) z_k^0, the annual rate of inspection of type k occupancies as reflected in historical data, (2) $z^1 = 2$ inspections per year (once each 6 months), and (3) $z^2 = 12$ inspections per year (once a month). Therefore, the estimators to be derived are $\hat{p}_{ik\ell t}(z_k^0)$, $\hat{p}_{ik\ell t}(z^1)$, and $\hat{p}_{ik\ell t}(z^2)$ for all i, k, ℓ, and t. These estimates are to be formed from subjective and objective data and are used as the basis of a piece-wise linear approximate function used in the planning models (i.e., the "effectiveness function").

Objective Estimates. Historical data were available to estimate p(z) at an inspection frequency of z_k^0. The data base was a two-year period of fires recorded in the Department's "log books." The extent of the data allowed estimates of p(z) to be made for both t = 1 and t = 2 and to be based on two repetitions of both six-month intervals. However, there was no way to associate a fire with a census tract so that only "city-wide" estimates could be made.

Estimates of p(z) for z^1 and z^2 are based on an experiment conducted by the Atlanta Fire Prevention Bureau. The experiment consisted of performing routine inspections in predetermined buildings. Some buildings are inspected once per six months (z^1), others once a month (z^2). The experiment is an on-going effort that is scheduled to continue for at least one year. However, results of the first six months (i.e., the first "phase" of the experiment) were used to establish initial objective estimates of $p(z)$ to be used in obtaining initial solutions to the planning problems confronting the Bureau.

All 20 of the full-time inspectors in the Bureau were used to conduct the experiment. However, this relatively limited manpower forced the scope and results of the experiment to be somewhat narrow. Nevertheless, use of a controlled experiment rather than mere regression of historical inspection rates vs. fire incidents is essential to drawing meaningful conclusions and estimates. This is the only way to reflect true cause-effect relationships in the data analyzed.

The experimental "design" used is a single factor design in which effects of multiple factors are accounted for by repeating the design for each type fire. That is, each i-k-t tested constitutes a separate experiment. Unfortunately, not enough manpower was available to test all i-k-t categories for $i = 1,2,\ldots,118$, and $k = 1,2,\ldots,34$ as applies to Atlanta. Therefore, inspection categories were set up in which area effects were ignored and six classifications of occupancy type were used. The specific occupancy types that fall into these six occupancy categories are identified in Appendix I. The groupings were based on likely similarity of inspection effectiveness.

Letting $j = 1,2,\ldots, 6$ be the subscript for the new classification of the original $k = 1,2,\ldots,34$ occupancy types, the parameter to be estimated is

$p_{j\ell t}(z)$ = the probability of a type j structure having ℓ fires during six-month interval t when it is inspected at rate z.

Therefore, the objective estimate, $\hat{p}_{j\ell t}(z)$, is being used as the objective estimate for all areas (i) and all type structures grouped in category j. That is, in terms of objective data,

$$\hat{p}_{ik\ell t}(z) = \hat{p}_{j\ell t}(z)$$

for j such that k is in category j, and for all i. This holds for the estimation of $p(z_k^0)$ as well as $p(z^1)$ and $p(z^2)$.

The design and execution of the experiment proceeded as follows:

1. Stratified sampling was used to determine the sample size of each of the six categories. The city's "Plan File" was used to ascertain percentage of structures in each category assuming 3428 (see Appendix I) structures could be inspected.

2. Each sample was divided into two equal subsamples, one to be inspected at a rate of z^1 and the other at a rate of z^2.

3. Each of the 20 full-time inspectors participating in the experiment was assigned a random number of structures to inspect in each of the six categories at each rate (therefore, 12 samples). The randomization was subject to the requirement that the total of the samples assigned to inspectors be equal to the total sample sizes required.

4. Individual structures and the month in which they were to be inspected were selected randomly by each inspector, with the exception that they were instructed to select half the buildings in each sample with construction dates prior to 1953 and half later than 1953 (so that the influence of age of a structure might later be examined). Spot checks were conducted to ensure random selection on the part of the inspectors.

5. Each inspector conducted a routine inspection of the structures selected during the selected months. The inspections began in April, 1973, with the first six-month phase ending in September, 1973. The results (inspection date, violation found, etc.) were coded and keypunched for each inspection (see Appendix I for form used as well as other details of the experiment).

Concurrently with conducting and coding the inspections a computerized file of structural fires occurring during the experiment period was maintained (both the "Inspection File" and the "Fire Incident File" are intended to be permanent, on-going computerized data files). Together, these files provide the data used to establish the experimental estimates of p(z).

The analysis of the results of the experiment is based on four considerations. These are (1) the specific fires occurring during the experiment to count as part of the experimental data, (2) the structures to count as sample members, (3) the statistical significance of differences in the numerical results, and (4) the adequacy of the samples selected for each inspection category. Each of these considerations is discussed below.

1. *Counting Fires as "Successes."* The experimental results reflect a "transient" condition of the fire incidence process. This is due to the use of inspection rates during the experiment which were not in effect before the experiment began. The six-month experimental period is not sufficient to produce "steady-state" conditions reflecting the true effects of the experimental rates. This can allow the inspection policies and operations of the Bureau which were in practice before the experiment to bias the experimental results. Therefore, it is necessary to adjust these results to prevent the occurrence of such bias.

Adjustment of the experimental results entails determining which of the fires observed before, during, and after the experiment should be counted in determining "successful" events. (Recall that an event is the observation of one of the i-k structures at the end of the experiment, and a successful event occurs when ℓ fires are observed or counted in an i-k structure.) In order to eliminate the possible "transient" bias, not all fires observed can be counted.

The approach taken in this research is that the counting strategy used for the experiment should be consistent with the application of the results to actual operations. The experiment results will be used to estimate the expected number of fires in an area in a future six-month period. The total number of fires occurring during this specific six-month interval needs to be reflected in the experimental estimates. Therefore, the counting strategy must not count fires occurring before or after the six-month experimental interval. Only fires occurring during the interval should be counted.

However, not all fires occurring during the experiment period should be included in the count. Application of the experimental estimates will be to six-month periods in which the inspection policies are on-going and repeating. For example, if a structure is inspected under a policy of one inspection per six months, then during a two-year horizon of actual operation there will be four inspections in that structure, each six months apart. Each fire occurring during the two-year period in that building will have been affected by an inspection no further away than six months prior to the fire.

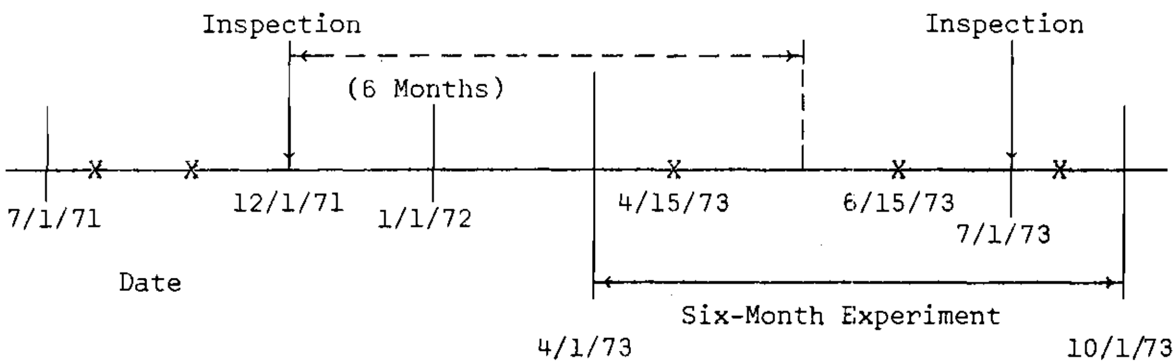
Initial experimental results are based on only one six-month interval. The inspection rates used during the experiment were not in effect before the experiment began. Therefore, counting all fires occurring during the experimental interval would not reflect an on-going, repeating nature of the inspection policies being tested. However, this condition or nature can be reflected in experiment results if only those fires occurring within 12/z months of an inspection are counted. This would allow each fire counted to have been subject to the effects of an inspection for the same length of time as would be the case in normal, routine operations. If this were not done, a fire could be counted against the effect of an inspection rate of, say, once each six months when, in fact, it had been three years since the most recent inspection in that building.

In terms of the two inspection policies being tested in the experiment, this counting strategy has the following interpretation. In samples inspected at z^1 (i.e., one inspection per six months), a fire occurring in a sampled building is counted if it occurred after the experimental inspection, or if it occurred before that experimental inspection *and* the building was inspected within a six-month period prior to the fire.

Similarly, in samples inspected at z^2 (one inspection per month), all fires occurring after inspections in the respective structures during the experiment period are counted. A fire occurring during the experiment period but before the respective building was inspected is still counted if it was inspected within one month prior to the fire.

2. *Determination of Sample Members.* Some fires occurring during the experiment in sampled buildings will not be counted due to the date

of the fire relative to the date of inspections in the building. When this happens, counting other fires occurring in these same buildings may produce misleading results in classifying the total number of fires occurring in those buildings in the six-month period (i.e., the value of "ℓ"). For example, suppose the inspection-fire history of a structure used in the experiment (inspected once per six months) was as follows:



where "X" implies a fire. In this example, even though three fires occur during the experiment period, it would not be accurate to classify this structure as being inspected at a rate of once per six months and as having three fires during the experiment. The fire occurring on 6/15/73 should not be counted against the effect of the first inspection and so should not be counted at all. However, it would not be accurate to consider this building (and the two fires counted in it) as a sample point in the group having $\ell = 2$ fires per six months. That is, the third fire not counted certainly influences the level of fires (ℓ) per six-month period that tends to occur in this structure. Therefore, since there is no proper manner in which to classify the "ℓ" level of this structure, it should not be counted in any of the ℓ samples in its j-t category.

In general, a structure chosen for the experiment will be excluded from the sample if any fire occurring during the experiment period in the structure is not counted for any of the reasons mentioned earlier. Therefore, the effective sample size for analysis purposes, $N_j(z)$, is the original number in the stratified sample minus the number excluded due to a fire in it not being counted (this is for all j-ℓ-t-z samples).

3. *Hypothesis Testing.* The estimates based on historical data and those based on the experiment must be examined to determine whether or not differences in the probability of fires at different inspection frequencies are statistically significant. $\hat{p}(z^0)$ must be compared to $\hat{p}(z^1)$. If $\hat{p}(z^0)$ is not statistically *greater* than $\hat{p}(z^1)$, the experiment results should be combined for z^0 and z^1. A similar comparison must be made for z^1 and z^2.

The procedure is to test the following hypothesis (illustrated for z^1 and z^2 only):

$$H_0: \hat{p}_{j\ell t}(z^1) = \hat{p}_{j\ell t}(z^2)$$

$$H_1: \hat{p}_{j\ell t}(z^1) > \hat{p}_{j\ell t}(z^2)$$

An exact test is necessary since $\hat{p}_{j\ell t}(z)$ is small and $N_j(z)$ is not large. Duncan [32] presents such a test. The procedure involves computation of the probability of observing the given sample results or "a result that deviates further from that expected on the basis of the given hypothesis (H_0)." If this probability is less than the level of significance, α, adopted for the test, the hypothesis is rejected in favor of H_1. This implies that $\hat{p}_{j\ell t}(z^1)$ is significantly greater than $\hat{p}_{j\ell t}(z^2)$ [since the test will be carried out only if the numerical value of $\hat{p}_{j\ell t}(z^1)$ is greater than that of $\hat{p}_{j\ell t}(z^2)$]. Consequently, we conclude that z^2 is more effective than z^1 for that j-ℓ-t category. If H_0 is not rejected, we conclude that there is no significant difference in the effectiveness of z^2 and z^1. In this case, $\hat{p}_{j\ell t}(z^1)$ and $\hat{p}_{j\ell t}(z^2)$ are combined to yield an estimate of $p_{j\ell t}$ at both z^1 and z^2.

The specific steps in the procedure are listed below. They must be repeated for each j-ℓ-t inspection category (recall that there are two samples for each j-ℓ-t category: one for structures inspected at z^1, and one for those j-ℓ-t structures inspected at z^2).

1. Compute the joint probability of observing the experimental results for the two j-ℓ-t samples, P(AB). That is, let

   A = the occurrence of $x^s_{j\ell t}(z^1)$ buildings of occupancy type j inspected at rate z^1 having ℓ fires during the *tth* six-month period (there are a possible $N_j(z)$ building for which this can happen).

   B = the occurrence of $x^s_{j\ell t}(z^2)$ buildings of occupancy type j inspected at rate z^2 having ℓ fires during six-month interval t.

   $x^{(1)} = x^s_{j\ell t}(z^1)$.

$$x^{(2)} = x^{s}_{j\ell t}(z^2).$$

$$x^{(T)} = x^{(1)} + x^{(2)}.$$

$$N^{(1)} = N_j(z^1).$$

$$N^{(2)} = N_j(z^2).$$

$$N^{(T)} = N^{(1)} + N^{(2)}.$$

$$p = p_{j\ell t}(z).$$

So that

$$P(AB) = \binom{N^{(1)}}{x^{(1)}}\binom{N^{(2)}}{x^{(2)}} p^{x^{(T)}} (1-p)^{N^{(T)}-x^{(T)}} .$$

2. Compute the probability of observing the *combined* sample results, P(C). That is,

$$P(C) = \binom{N^{(T)}}{x^{(T)}} p^{x^{(T)}} (1-p)^{N^{(T)}-x^{(T)}} .$$

3. Compute the conditional probability of observing event A, given that the combined sample results, C, were in fact realized. That is,

$$P(A/C) = P(AB)/P(C)$$

(Note that the joint event AB is, in this case, equivalent to the joint event AC.)

so that

$$P(A/C) = \binom{N^{(1)}}{x^{(1)}} \binom{N^{(2)}}{x^{(2)}} \bigg/ \binom{N^{(T)}}{x^{(T)}}.$$

4. If $P(A/C) \geq \alpha$, accept H_0. If not, proceed.

5. Compute the probability of observing $x^{(2)} - 1$ simultaneous with $x^{(1)} + 1$ in the sample of $x^{(T)}$ "counts." That is, compute

$$\bar{P}(A/C) = \binom{N^{(1)}}{\bar{x}^{(1)}} \binom{N^{(2)}}{\bar{x}^{(2)}} \bigg/ \binom{N^{(T)}}{x^{(T)}}$$

where $\bar{x}^{(1)} = x^{(1)} + 1$, $\bar{x}^{(2)} = x^{(2)} - 1$.

6. Compute the cumulative probability of observing $(x^{(1)},x^{(2)})$ counts and $(\bar{x}^{(1)},\bar{x}^{(2)})$ counts. That is,

$$P^T(A/C) = P(A/C) + \bar{P}(A/C).$$

If $P^T(A/C) \geq \alpha$, accept H_0. If not, repeat steps (5) and (6) by incrementing the step increase in $\bar{x}^{(1)}$ and the step decrease in $\bar{x}^{(2)}$ until either $P^T(A/C) \geq \alpha$ or $\bar{x}^{(2)} = 0$. If the latter contingency occurs, reject H_0 and conclude that $\hat{p}_{j\ell t}(z^1) > \hat{p}_{j\ell t}(z^2)$. Otherwise, accept H_0.

4. *Adequacy of Sample Size.* At the end of the initial six-month experiment period (September 30, 1973) sample results were analyzed to determine whether or not additional observation time was necessary.

A basic reason for continuing the experiment for a second six-month period is to obtain objective estimates for $p_{j\ell t}(z)$ for $t = 2$ (the second six-month interval during a year, October-March). However, a second reason would be to increase the sample size for j-ℓ-t-z categories for which the single $t = 1$ sample is not "adequate."

A sample is considered to be adequate if it produces an estimator, $\hat{p}_{j\ell t}(z)$ for z^0, z^1, or z^2, which is "acceptable." Two criteria are used to establish the acceptance of estimators. The first is based on an analysis of the confidence interval containing $p_{j\ell t}(z)$. If the width of the 90 per cent confidence limit (based on the sample results) about $p_{j\ell t}(z)$ is less than or equal to the width considered by Fire Department Management as being acceptable, W^c, then the sample estimator is said to be acceptable. If not, the estimator cannot be used and additional data must be added to the sample (which may require additional six-month experiments).

The value of the acceptable confidence interval width, W^c, should reflect the consequences and costs of incorrect estimates. Such costs depend on the uses made of the estimates. There are many applications of the estimates $\hat{p}(z)$ in fire department planning, including determination of inspection rates and inspection districts, determining inspection schedules, and locating fire stations. If there were only one application, the sensitivity of decisions and costs of erroneous decisions could be examined as a function of changes in $\hat{p}(z)$. Tradeoffs in these costs and in the costs of additional sampling could be examined to determine a "best" value of W^c. Unfortunately, this is not the case. There appears to be no practical way to quantitatively evaluate the relative costs of changes in $\hat{p}(z)$ in all its applications.

The approach used in this research is to base the value of W^c on a subjective evaluation of the consequences of an incorrect $\hat{p}(z)$ versus the required increase in samples sizes to improve the accuracy of $\hat{p}(z)$. Insight into the magnitude of an appropriate W^c was gained by examining the actual confidence interval widths for samples from both historical data and the experiment. The specific value of W^c used is given in Appendix II. It is acknowledged that this value is somewhat arbitrary, but there does not appear to be a more suitable approach.

An exact confidence limit must be developed since for the small value of the parameter ($\leq .01$) and relative small sample sizes (31, 150, etc.) normal or poisson approximations are, in general, not adequate ([33], pp. 399-405).

The exact approach used in this research follows the procedures outlined by Cowden ([33], pp. 105-108). The end results are two values of $p_{j\ell t}(z)$, say $\hat{p}^1_{j\ell t}(z)$ and $\hat{p}^2_{j\ell t}(z)$, such that

$$P\left(\hat{p}^1_{j\ell t}(z) \leq p_{j\ell t}(z) \leq \hat{p}^2_{j\ell t}(z)\right) = .90,$$

The two limiting values of p are based on the sample results, say $x^s_{j\ell t}(z)$ fires in the sample taken of $N_j(z)$ structures for category j-ℓ-t-z.

The lower limit, $\hat{p}^1$, is established by finding a value of p for which $P(x \geq x^s/p) = .05$, or, alternatively, $P(x \leq x^s-1/p) = .95$. That is, find $\hat{p}^1_{j\ell t}(z) = p$ such that

$$\sum_{x=0}^{x^1} \binom{N}{x} p^x (1-p)^{N-x} = .95,$$

where $x^1 = x^s_{j\ell t}(z) - 1$, $N = N_j(z)$.

The upper limit, $\hat{p}^2$, is established by finding a value of p for which $P(x \le x^s/p) = .05$. That is, find $p^2_{j\ell t}(z) = p$ such that

$$\sum_{x=0}^{x^2} \binom{N}{x} p^x (1-p)^{N-x} = .05,$$

where $x^2 = x^s_{j\ell t}(z)$, $N = N_j(z)$.

The width of this interval is $W_{j\ell t}(z) = \hat{p}^2_{j\ell t}(z) - \hat{p}^1_{j\ell t}(z)$. If $W_{j\ell t}(z) \le W^c$, then the 90 per cent confidence limit for that j-ℓ-t-z category is sufficiently small and the sample is therefore acceptable. In this case, $\hat{p}_{j\ell t}(z)$ is judged to be a significant estimator of $p_{j\ell t}(z)$ and is used in the planning models as the objective estimate of p for that category.

If $W_{j\ell t}(z) > W^c$, then the interval is not small enough. In this case, $\hat{p}_{j\ell t}(z)$ is not judged to be a significant estimator of $p_{j\ell t}(z)$ so that no objective estimate is available (at this time). A larger sample for this j-ℓ-t-z category is needed to yield an acceptable objective estimate. This requires additional six-month experiment periods for six-month interval t.

In the meantime, some estimate of p is needed for inadequate

j-ℓ-t-z samples. Estimates of p for all j-ℓ-t-z categories are needed for the initial solution of the planning models. Therefore, the subjective estimates obtained from management are used as the total estimate of $p_{j\ell t}(z)$ for inadequate j-ℓ-t-z categories.

The second criterion used to judge the acceptability of sample estimators concerns the relative magnitude of $p(z^0)$, $p(z^1)$, and $p(z^2)$. There is no theoretical reason that $p(z^0) < p(z^1) < p(z^2)$. That is, $p(z)$ is a nonincreasing function. However, due to the influence of random factors, the samples taken from historical data and in the six-month experiment may result in estimators for a j-ℓ-t category which violate the relation $p_{j\ell t}(z^0) \geq p_{j\ell t}(z^1) \geq p_{j\ell t}(z^2)$. For such categories, the samples are not large enough to smooth out the extraneous random influences. Such samples are combined to produce a single estimate for each z level violating the relationship.

The numerical results of (1) counting fires, (2) testing hypotheses, and (3) testing sample adequacy are given in Appendix II for the Atlanta application.

Subjective Estimates

In addition to the objective estimates of $p(z)$ derived from historical and experimental data, subjective estimates were obtained. There are two basic reasons for using subjective estimates. First, the samples for several inspection categories (j-ℓ-t-z combinations) in the six-month experiment are inadequate. That is, they are not large enough to provide "acceptable" estimates of $p(z)$. Further, there is currently no objective sample data for $t = 2$ (i.e., October-March) for

z^1 or z^2 in any j-ℓ category. (These data will be forthcoming as the Department continues the experiment.)

This lack of sufficient experimental data necessitates reliance on supplemental information in order that estimates of $p(z)$ be obtained for all j-ℓ-t-z categories. Subjective estimates can be used for this purpose. However, as the experimentation continues beyond the first six-month interval, the "adequacy" of objective samples will be enhanced. This will allow more weight to be placed on the objective estimates than can be done initially.

A second reason for use of subjective estimates is that the ultimate estimate of $p(z)$ should actually be a *forecast* of p in the future time period in which the planning models will be used. The objective estimates necessarily reflect conditions prevailing at the time of the experiments. Use of these objective estimates alone would not allow *future* conditions (changes in neighbor awareness, neighbor socio-economic make-up, building codes, etc.) that have changed since the experiment period to be accounted for. Similarly, factors that were altogether extraneous to the experiment can be accounted for. For example, if an intense fire preventure lecture series is anticipated in the future, its effect on the expected number of fires can be estimated through the subjective estimator of $p(z)$.

Subjective estimates were obtained for $k = 1,2,\ldots,34$ classifications of occupancy type rather than for the $j = 1,2,\ldots,6$ classifications used in the experiment. More categories could be estimated subjectively than in the experiment. The 34 classifications were derived by Bureau management from their 66 primary occupancy type categories. The groups in the reduced classification reflect similarities in (1) possible effects of increases in inspection frequency, (2) potential seriousness of fires, (3) the time per square foot to inspect, and (4) past inspection frequency (z_k^0).

"City-wide" estimates were made rather than making separate estimates for each area, i. Bureau management felt that the effectiveness of alternative inspection frequencies would be the same in all parts of the city.

Actual soliciting of the subjective estimates was a time-consuming, "pains-taking" process. Group meetings were held intermittently over a three-day period with management personnel (the Assistant Fire Marshall and two Supervisors). A large amount of time was spent on finding the most appropriate manner of questioning and response so that questions and answers would have meaning to the group.

The approach used was to provide the group with a numerical "base point" from which specific changes could be estimated. The base point was the number of structures in a group of 1000 of each occupancy category ($k=1,2,\ldots,34$) having "ℓ" fires in a six-month period, given that they were inspected at a rate of z_k^0. The two-year sample of historical data was used for these numbers.

Using these figures as an indication of the effectiveness of z_k^0, a consensus was obtained from the group as to the percentage change in the number of structures having "ℓ" fires for z^1 and z^2. This was done for each k-ℓ-t combination. From this information, subjective estimates can be obtained of $p_{k\ell t}(z)$ for z^1 and z^2. Bureau Management did not feel that there was any reason to change the $\hat{p}_{k\ell t}(z^0)$ derived from historical data and used as the base point. They felt that these figures would reflect conditions in the next year.

The accuracy of subjective estimates is always questionable. However, the men who provided this information for the Atlanta application have a combined experience in fire department operations of over 80 years. This includes more than 50 years in fire prevention work. This group expressed modest confidence in the reliability in the estimates they made.

Blending Objective and Subjective Estimates

The objective and subjective estimates of p(z) must be combined into a single estimator of $p_{ik\ell t}(z)$ for z^0, z^1, and z^2. Estimates derived from historical data for k-ℓ-t-z_k^0 combinations, and those derived from experimental results for j-ℓ-t-z for $t = 1$ and $z = z^1$ and $z = z^2$, are used to obtain the composite estimate for each i-k-ℓ-t-z combination.

The rationale used in this research to combine the two types of estimates for each i-k-ℓ-t-z is to use a Bayesian approach. The subjective estimate is the "prior" and the objective estimate is the "sampling" estimate. However, due to the fact that some samples (for j-ℓ-t-z categories) in the experiment may be inadequate, an objective estimate will not exist for every i-k-ℓ-t-z combination. When this occurs, no weight will be given to the objective estimate in the Bayes equation, and the composite estimator will be composed of the subjective value only.

The procedure for combining the two estimates can be described using the following notation:

$\hat{p}^{p}_{ik\ell t}(z)$ = the subjective estimate of p(z) for type k structures in area i to have ℓ fires in six-month interval t when the inspection rate is z.

$\hat{p}^{s}_{ik\ell t}(z)$ = the objective estimate of p(z) for i-k-ℓ-t-z.

$\hat{p}_{ik\ell t}(z)$ = the combined estimate of p(z) for i-k-ℓ-t-z.

$\hat{p}^{p}_{k\ell t}(z)$ = the subjective estimate of p(z) obtained directly from Bureau Management for k-ℓ-t and z^1 and z^2.

$\hat{p}^{s}_{j\ell t}(z)$ = the objective estimate of p(z) obtained from experimental results for j-ℓ and t = 1 and z^1 and z^2; from subjective estimates for j-ℓ and t = 2 and z^1 and z^2; and from historical results for j-ℓ-t and z^0_k.

Recall that k = 1,2,...,34, and j = 1,2,...,6, and that $\hat{p}(z)$ is obtained for only three values of z - z^0_k, z^1, and z^2.

Estimates for each i-k-ℓ-t-z combination are the ultimate goal of the estimation procedure. Estimates had to be made directly for k-ℓ-t-z and j-ℓ-t-z combinations rather than i-k-ℓ-t-z. The relationship between the categories of estimates is

$$\hat{p}^{p}_{ik\ell t}(z) = \hat{p}^{p}_{k\ell t}(z) \qquad \text{for all } i,$$

and

$$\hat{p}^{s}_{ik\ell t}(z) = \hat{p}^{s}_{j\ell t}(z) \qquad \text{for all } i, \text{ and for } j \text{ such that } k \text{ is in the } j\textit{th} \text{ category of occupancy types.}$$

Recall that $\hat{p}(z)$ follows a binomial distribution. Therefore, $\hat{p}^{s}(z)$ also follows a binomial distribution with parameter p(z). Assuming that the prior distribution of $\hat{p}^P(z)$ is a beta distribution with parameters $r = 1000p(z)$ and $n = 1000$, then the posterior distribution of $\hat{p}(z)$ has a mean of [34]

$$\hat{p}_{ik\ell t}(z) = \frac{x^S_{ik\ell t}(z) + x^P_{ik\ell t}(z)}{n^S_{ik\ell t}(z) + n^P_{ik\ell t}(z)}$$

where

$x^S_{ik\ell t}(z) = N_j(z)\hat{p}^S_{ik\ell t}(z)$ for j such that k is in category j.

$x^P_{ik\ell t}(z) = 1000\hat{p}^P_{ik\ell t}(z)$.

$n^S_{ik\ell t}(z) = N_j(z)$ for j such that k is in category j.

$n^P_{ik\ell t}(z) = 1000$ (since the sample size used in soliciting all subjective estimates was 1000 structures).

Or, equivalently,

$$\hat{p}_{ik\ell t}(z) = \left(\frac{N_j(z)}{N_j(z)+1000}\right)\hat{p}^S_{ik\ell t}(z) + \left(\frac{1000}{N_j(z)+1000}\right)\hat{p}^P_{ik\ell t}(z).$$

This Bayes equation assumes both $\hat{p}^S(z)$ and $\hat{p}^P(z)$ exist. However, as pointed out earlier, $\hat{p}^S(z)$ will not exist for inadequate j-ℓ-t-z categories, and no weight can be given to $\hat{p}^S(z)$ in this case. Therefore, let

$$\hat{p}_{ik\ell t}(z) = (1-w_k)\hat{p}^S_{ik\ell t}(z) + w_k\hat{p}^P_{ik\ell t}(z),$$

where

$$w_k = \begin{cases} 1 & \text{if the corresponding } j\text{-}\ell\text{-}t\text{-}z \text{ sample is not adequate,} \\ \left(\dfrac{1000}{N_j(z)+1000}\right) & \text{for } j \text{ such that } k \text{ is in category } j\text{, otherwise.} \end{cases}$$

A yearly estimate of $p(z)$ is needed in the planning models. Therefore, estimates for $t = 1$ and $t = 2$ must be combined. Let

$$\hat{p}_{ik\ell}(z) = \hat{p}_{ik\ell 1}(z) + \hat{p}_{ik\ell 2}(z).$$

Effectiveness Function

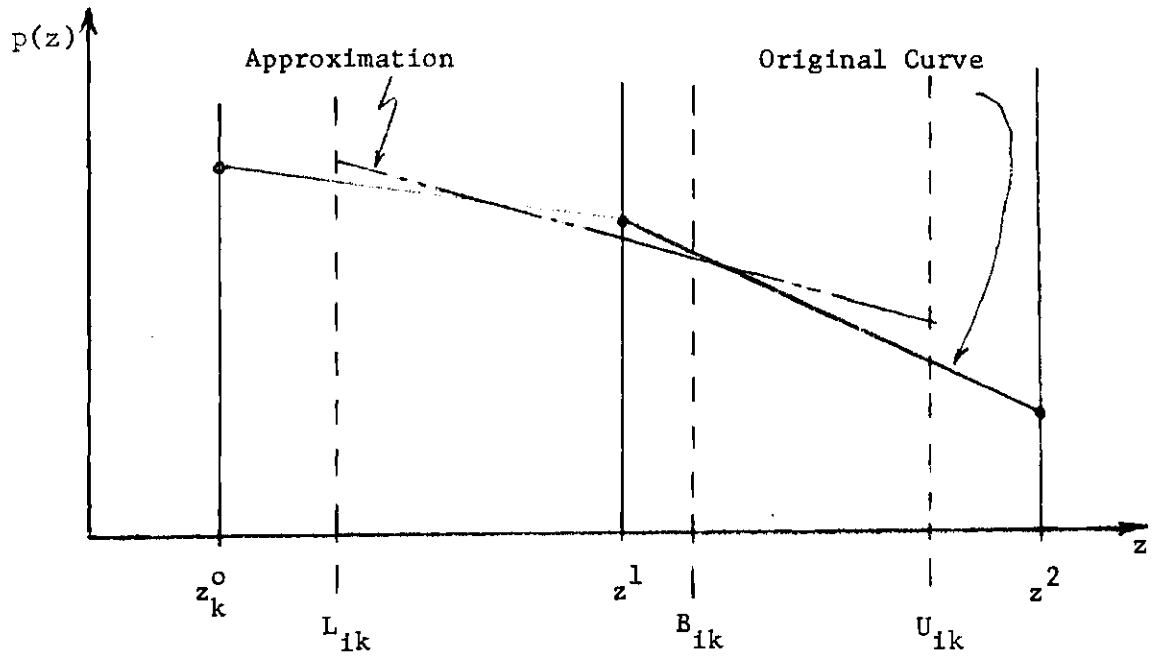
The estimate of p formulated above is a function of the inspection frequency z. The resulting estimation function, $\hat{p}(z)$, is an "effectiveness" or "production" function. There is little theoretical basis for the specific form of this function, other than it should be nonincreasing. There is no reason to believe that an increase in inspection frequency would cause an increase in the probability of a fire. As mentioned earlier, this reasoning is reflected in the second criteria used to judge the adequacy of sample results.

The numerical version of $\hat{p}(z)$ is obtained from the three data points $\hat{p}(z^0)$, $\hat{p}(z^1)$, and $\hat{p}(z^2)$ for each i-k-ℓ combination. The resulting curve will be nonincreasing since the second criteria of sample adequacy is incorporated in the definition of $\hat{p}_{ik\ell}(z)$. Obviously, more than three data points would be desirable in order to obtain a more accurate $\hat{p}(z)$. Manpower limitation prevented more points from inclusion in the experiment. However, the frequencies included between z_k^0 and z^2 cover the frequencies of interest in the planning problems. This is due to fairly tight upper and lower bounds on the allowable inspection rates in the Atlanta application.

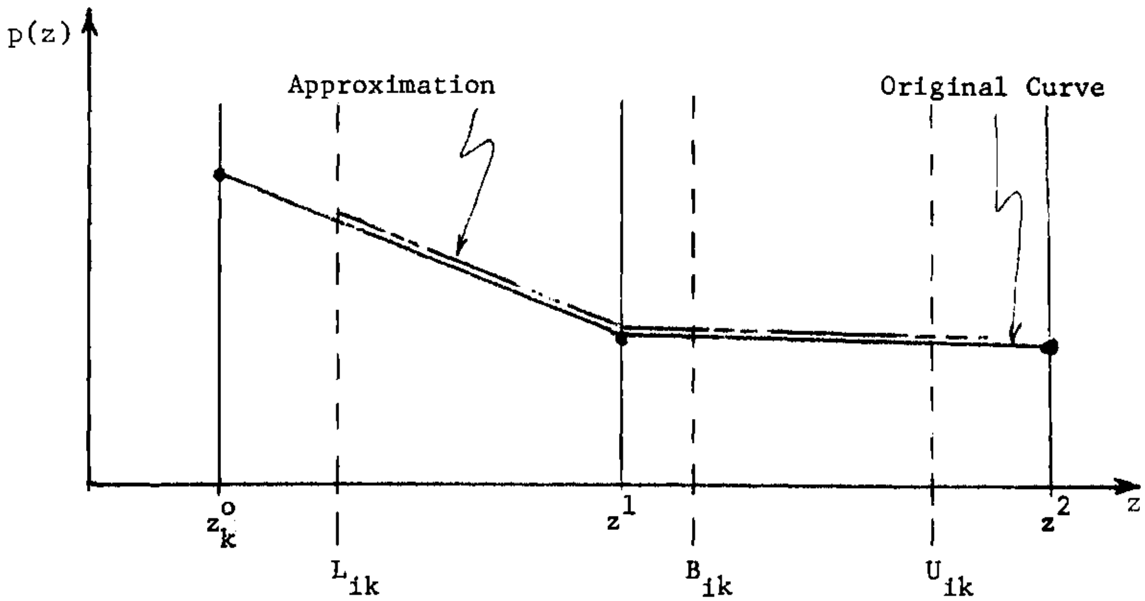
The numerical form of $\hat{p}(z)$ may be convex or concave. A convex function greatly enhances the ability to solve the planning problem in which $\hat{p}(z)$ is used to quantify the objective function. Minimization of a nonconvex function is well known to be a difficult problem. Therefore, the approach used in this research is to derive a convex approximation to the original $\hat{p}(z)$. In addition to being convex and nonincreasing, the approximation will be piece-wise linear. This new approximate function is used as the effectiveness function, $\hat{p}(z)$, in solving the planning problems.

The piece-wise approximation consists of two linear segments. They are defined from the lower limit, L_{ik}, to the upper limit, U_{ik}, for each i-k-ℓ function. The first segment is defined over the interval $[L_{ik},B_{ik}]$, and the second over $(B_{ik},U_{ik}]$, where B_{ik} is the integer midpoint between L_{ik} and U_{ik}.

If the original $\hat{p}(z)$ function is *nonconvex,* a linear, nonincreasing regression line is used as the approximation over the entire interval $[L_{ik},U_{ik}]$. In this case, both piece-wise segments have the same slope. If the original function is *convex,* the original curve itself or its direct extension is used as the piece-wise approximation. In this case, the slope over the first segment, $s_{ik\ell 1}$, may not be equal to the slope over the second segment, $s_{ik\ell 2}$. Two examples of this piece-wise approximation procedure are given in Figure 3.



Piece-Wise Approximation for Nonconvex Curve



Piece-Wise Approximation for Convex Curve

Figure 3. Examples of Pieces-Wise Linear Approximations of Effectiveness Functions

The resulting approximate effectiveness function is

$$\hat{p}_{ik\ell}(z) = \begin{cases} s_{ik\ell 1} z & \text{if} \quad z \in [L_{ik}, B_{ik}] \\ s_{ik\ell z} z & \text{if} \quad z \in (B_{ik}, U_{ik}] \end{cases}$$

where

$$B_{ik} = [(U_{ik} - L_{ik})/2 + L_{ik}] \quad \text{(the lowest integer value).}$$

The derivation of $s_{ik\ell 1}$ and $s_{ik\ell 2}$ is as follows.

Let $\hat{p}(z)$ = the original estimate, $\hat{p}_{ik\ell}(z)$.

<u>If $\hat{p}(z)$ is convex</u>,

let, for each i-k-ℓ,

$$r^{(1)} = \left(\hat{p}(z_k^0) - \hat{p}(z^1)\right)/(z^1 - z_k^0),$$

$$r^{(2)} = \left(\hat{p}(z^1) - \hat{p}(z^2)\right)/(z^2 - z^1)$$

and

$$p^{(1)} = \begin{cases} \hat{p}(z_k^0) - (L_{ik} - Z_k^0) r^1 & \text{if} \quad L_{ik} \le z^1 \\ \hat{p}(z^1) - (L_{ik} - Z^1) r^2 & \text{if} \quad z^1 < L_{ik} \le z^2 \end{cases}$$

$$p^{(2)} = \begin{cases} \hat{p}(z_k^0) - (B_{ik} - Z_k^0) r^{(1)} & \text{if } B_{ik} \le z^1 \\ \hat{p}(z^1) - (B_{ik} - Z^1) r^{(2)} & \text{if } Z^1 B_{ik} \le z^2 \end{cases}$$

$$p^{(3)} = \begin{cases} \hat{p}(z_k^0) - (U_{ik}-z_k^0)r^{(1)} & \text{if } U_{ik} \le z^1 \\ \hat{p}(z^1) - (U_{ik}-z^1)r^{(2)} & \text{if } z^1 < U_{ik} \le z^2. \end{cases}$$

Observe that $p^{(1)} = \hat{p}_{ik\ell}(L_{ik})$, $p^{(2)} = \hat{p}_{ik\ell}(B_{ik})$, and $p^{(3)} = \hat{p}_{ik\ell}(U_{ik})$. Also, note that z_k^0 may be less than z^1 for some k and greater than z^1 for other k. The above derivation assumes that $z_k^0 \le z^1$. If $z_k^0 > z^1$, the role of z_k^0 and z^1 must be reserved in all calculations.

<u>If $\hat{p}(z)$ is nonconvex,</u>

let $z^m = z_k^0$ for m = 1, z^1 for m = 2, and z^2 for m = 2.

Then,

$$a = \sum_{m=0}^{2} \hat{p}(z^m)/3$$

$$b = \sum_{m=0}^{2} (z^m-\bar{z}^m)\hat{p}(z^m)/ \sum_{m=0}^{2} (z^m-\bar{z}^m)^2,$$

and

$$p^{(1)} = a + bL_{ik}$$

$$p^{(2)} = a + bB_{ik}$$

$$p^{(3)} = a + bU_{ik}.$$

In both cases, the slopes of the two segments are

$$s_{ik\ell 1} = (p^{(2)}-p^{(1)})/(B_{ik}-L_{ik})$$

and

$$s_{ik\ell 2} = (p^{(3)}-p^{(2)})/(U_{ik}-B_{ik})$$

A summary of the steps involved in the procedure used to determine the effectiveness function, $\hat{p}_{ik\ell}(z)$, is given in Figure 4. Note that the steps presented are based on the analysis of the first six-month experimental period, historical data, and subjective estimates.

The numerical values of $s_{ik\ell 1}$ and $s_{ik\ell 2}$ for the Atlanta application are given in Appendix II. Note that the values given are independent of the area of location. This results from the fact that the inspection limits, L_{ik} and U_{ik}, are independent of location; that is, only L_k and U_k are used.

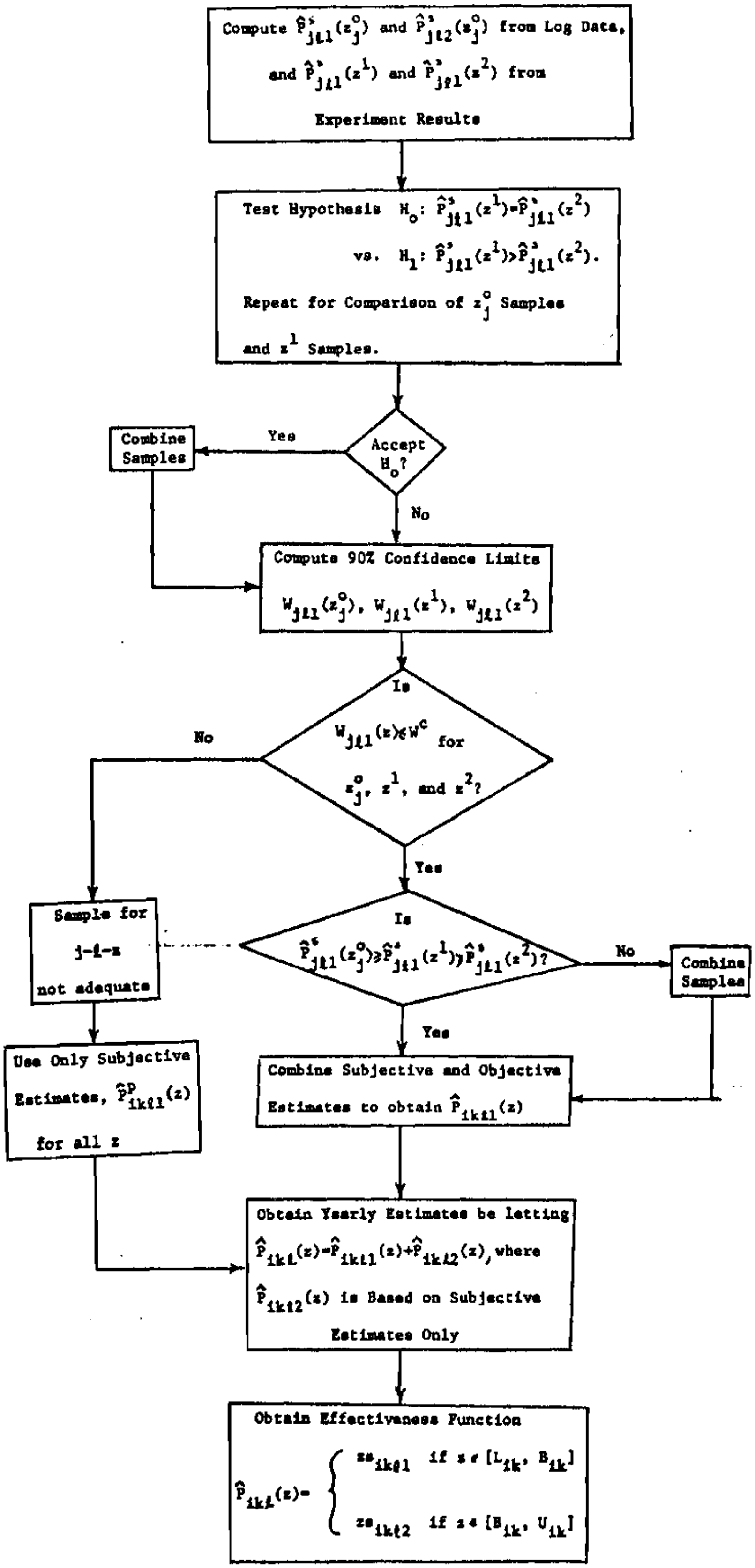


Figure 4. Determination of Effectiveness Function

CHAPTER V

DETERMINATION OF INSPECTION RATES AND INSPECTION DISTRICTS

As discussed in Chapter III, the planning problems of determining inspection frequencies and determining inspection districts are analyzed "simultaneously." That is, because of the interrelationships between the two problems, they are both included in the same mathematical model. The scheduling problems discussed in Chapter III are treated in separate models. In this chapter, analysis of the "rate-district" problem is described. The discussion includes the formulation of a mathematical model and a procedure for obtaining a solution. A similar description of the analysis of the scheduling problems will be given in Chapter VI.

Model

Modelling Approach

There are four criteria involved in the combined problem of determining inspection rates and inspection districts. They are (1) the expected potential seriousness of building fires, (2) the workload balance among inspection districts, (3) the compactness of districts, and (4) the contiguity of districts. The first criterion is associated with the "Rate Determination" problem, while the other three are part of the "District Determination" problem, both being described in Chapter III.

If all four criteria were treated explicitly in the objective function of the combined problem, weights would have to be given to the relative "value" of each criterion. This would require Bureau Management to assess explicit relative values for each criterion. Unfortunately, little is known about the relationship between these criteria and their relative value. However, the objective of reducing the expected potential seriousness of fires is thought by Bureau Management to be more important than any of the three criteria associated with the districting problem.

The approach used here is to include only the rate problem criterion of expected potential seriousness of fires in the objective function. The three criteria of the district problem are treated as constraints. In this approach, management must place a limit on the allowable level of each of the three district problem criteria.

Notation

Make the following definitions.

Decision Variables

z_{ik} = the number of scheduled, routine inspections conducted during the year in occupancy type k in census tract i ($k=1,2,\ldots,k$; $i=1,2,\ldots,I$).

$\underline{z}_{i.}$ = the vector $(z_{i1}, z_{i2}, \ldots, z_{ik})$.

$\underline{z}$ = the vector $(\underline{z}_{1.}, \underline{z}_{2.}, \ldots, \underline{z}_{I.})$.

y_{ij} = 1 if census tract i is assigned to district j; 0, otherwise ($j=1,2,\ldots,J$).

$\{S_j\}$ = the set of census tracts, i, which constitute district j.

Functions

$V_{ik}(z_{ik})$ = the "value" of inspecting all i-k structures (i.e., all type k occupancies in area i) when the rate of scheduled, routine inspections is z_{ik}.

$E_{ik}(f/z_{ik})$ = the expected number of fires in i-k structures in the year, given inspection rate z_{ik}.

$p_{ik\ell}(z_{ik})$ = the probability of a type k structure in area i having ℓ fires in the year, given that the i-k structure is inspected at a rate of z_{ik}.

$\hat{p}_{ik\ell}(z_{ik})$ = the piece-linear and convex approximation to $p_{ik\ell}(z_{ik})$.

$T_i(\underline{z}_{i.})$ = the man-hours required by all fire prevention operations during the year in area i, given inspection rates $\underline{z}_{i.}$.

$T_{ik}^{(1)}(z_{ik})$ = the hours (or man-hours) required per year for scheduled, routine inspections in all i-k structures, given z_{ik}.

$T_{ik}^{(2)}(z_{ik})$ = the hours required per year for nonscheduled, routine inspections in all i-k structures, given z_{ik}.

$T_{ik}^{(3)}(z_{ik})$ = the hours required per year for reinspections in all i-k structures.

$T_i^{(4)}(\underline{z}_{i.})$ = the hours required per year for fire report investigations in area i, given $\underline{z}_{i.}$.

$T_i^{(5)}(\underline{z}_{i.})$ = the hours required per year for indirect fire prevention work in area i.

$T^{(6)}(\underline{z})$ = the hours required per year for indirect work city-wide.

Parameters

N_{ik} = the number of i-k structures.

L_{ik} = the lower limit on the allowable level of z_{ik}.

U_{ik} = the upper limit on the allowable level of z_{ik}.

B_{ik} = the integer midpoint (smallest integer value of continuous midpoint) of the interval $[L_{ik},U_{ik}]$.

α_{ik} = the time (hours) required to conduct a routine inspection of an i-k structure (α_{ik} is the average of the time to inspect all individual structures in area i of occupancy type k).

β_{ik} = the percentage of the N_{ik} structures which require nonscheduled, routine inspections during the year.

r_{ik} = the annual number of reinspections i n all i-k structures.

γ_i = the time required to investigate a fire report in area i.

b_i = the number of fire reports which must be investigated in area i during the year.

ϕ_{ik1} = the time (hours) required to conduct "Gas and Oil," "Bottle Gas," or other nonstructural inspection (as part of indirect work duties) in occupancy type k in area i.

h_{ik1} = the number of nonstructural inspection per year in occupancy type k in area i.

ϕ_{ik2} = the time required to answer a "complaint" from an occupant of a type k occupancy in area i.

h_{ik2} = the annual number of complaints in type i-k structures.

ϕ_{i3} = the time required to conduct a "new building" inspection in area i.

h_{i3} = the annual number of "new building" inspections required in area i.

h_4 = the time (hours) required each year for indirect activities of court actions, fire drill supervision, radio and TV talks, photo-work, school and public education, and inspector self-education.

θ_{ik} = the time (hours) required to conduct a reinspection in an i-k structure.

$\{A_i\}$ = the set of areas which are adjacent to area i.

d_{ij} = the distance between the center of gravity (with respect to workload) of area i and the center of gravity of district j.

M = the number of man-hours available to conduct fire prevention operations during the year.

C = the allowable compactness of any district (C is expressed in units of "work hours-distance units squared").

Δ = the allowable percentage deviation of the workload of any district from the average district workload.

v_{ik} = the potential seriousness or hazards of a fire in an i-k structure.

P = the number of years in planning horizon for decisions on inspection rates, districts, and schedules.

Using the above notation, the "work-time" functions, $T^{(\cdot)}(\cdot)$, are being formulated as follows:

$$T_{ik}^{(1)}(z_{ik}) = \alpha_{ik}N_{ik}z_{ik},$$

$$T_{ik}^{(2)}(z_{ik}) = \beta_{ik}\alpha_{ik}N_{ik},$$

$$T_{ik}^{(3)}(z_{ik}) = \theta_{ik}r_{ik},$$

$$T_{i}^{(4)}(\underline{z}_{i\cdot}) = \gamma_i b_i,$$

$$T_{i}^{(5)}(\underline{z}_{i\cdot}) = \sum_k (\phi_{ik1}h_{ik1}+\phi_{ik2}h_{ik2}) + \phi_{i3}h_{i3},$$

and

$$T_i(\underline{z}_i) = \sum_k\left(T_{ik}^{(1)}(z_{ik})+T_{ik}^{(2)}(z_{ik})+T_{ik}^{(3)}(z_{ik})\right) + T_i^{(4)}(\underline{z}_{i\cdot}) + T_i^{(5)}(\underline{z}_{i\cdot}).$$

It is convenient to group the "work-time" functions which are independent of z_{ik} together $\left(\text{except for } T^{(6)}(\underline{z})\right)$ for modelling purposes. Therefore, let

$$T_i = \sum_k\left(T_{ik}^{(2)}(z_{ik})+T_{ik}^{(3)}(z_{ik})\right) + T_i^{(4)}(\underline{z}_i) + T_i^{(5)}(\underline{z}_i)$$

or,

$$T_i = \sum_k (\beta_{ik}\alpha_{ik}N_{ik}+\theta_{ik}r_{ik}+\phi_{ik1}h_{ik1}+\phi_{ik2}h_{ik2}) + \gamma_i b_i + \phi_{i3}h_{i3}.$$

Mathematical Model

In this section, a statement of the decision problem will be given and the mathematical representation of each component of the problem will be presented as it is described. Then the objective function and constraints will be put together to form the mathematical programming model being considered. Finally, the model will be rewritten in a form that allows a piece-wise approximation of p(z) to be used in the objection function.

The decision problem is given in the following statement.

Find: (1) the number of scheduled, routine inspections to conduct during the year in each occupancy type in each area, z_{ik}, and (2) the combination of areas to form each district, y_{ij}.

Such that: the total "value" of scheduled, routine inspections is maximized, or, alternatively, that the expected potential seriousness of fires as affected by scheduled, routine inspections is minimized. The mathematical representation of the total "value" is

$$\sum_i \sum_k V_{ik}(z_{ik}) = \sum_i \sum_k v_{ik}E_{ik}(f/z_{ik})$$

$$= \sum_i \sum_k N_{ik} \Big(\sum_\ell \ell N_{ik} p_{ik\ell}(z_{ik}) \Big)$$

Therefore, the objective function of the model is

$$\underset{\underline{z},\underline{y}}{\text{Minimize}} \sum_i \sum_k v_{ik} N_{ik} \sum_\ell \ell p_{ik\ell}(z_{ik}).$$

In addition, the decision variables must meet the following constraints.

1. The total number of man-hours consumed by fire prevention operations must not exceed the man-hours available. In mathematical terms,

$$\sum_i \Big(\sum_k \alpha_{ik} N_{ik} z_{ik} + \beta_{ik} \alpha_{ik} N_{ik} + \theta_{ik} r_{ik} + \phi_{ik1} h_{ik1}$$

$$+ \phi_{ik2} h_{ik2}) + \gamma_i b_i + \phi_{i3} h_{i3} \Big) + h_4 \leq M,$$

or, equivalently,

$$\sum_i \Big(\sum_k \alpha_{ik} N_{ik} z_{ik} + T_i \Big) \leq M - h_4.$$

2. The frequencies of scheduled, routine inspections must not exceed their upper and lower limits. Mathematically,

$$L_{ik} \leq z_{ik} \leq U_{ik} \qquad \text{for all } i,k.$$

3. Inspection categories having the same probability of potential seriousness and the same upper and lower limits should also have the same frequency of scheduled, routine inspections. Mathematically,

$$z_{ik} = z_{i'k'} \quad \text{for all i-k and i'-k'}$$

combinations such that

$$v_{ik} \sum_{\ell} \ell p_{ik\ell}(z_{ik}) = v_{i'k'} \sum_{\ell} \ell p_{i'k'}(z_{i'k'}),$$

and

$$L_{ik} = L_{i'k'} \quad \text{and} \quad U_{ik} = U_{i'k'}.$$

(Note that $\sum_{\ell} p_{ik\ell}(z_{ik})$ is the probability of realizing at least one fire in i-k.)

4. The frequency of scheduled, routine inspections is an integer multiple of 1/P. Mathematically,

$$Pz_{ik} \equiv 0 \pmod 1 \quad \text{for all i,k.}$$

5. The workload deviation or balance of each district formed is within the upper and lower acceptance limits. Mathematically,

$$(1-\Delta)\bar{T} \le \sum_i \Big(\sum_k \alpha_{ik} N_{ik} z_{ik} + T_i\Big) y_{ik} \le (1+\Delta)\bar{T}$$

where $\bar{T} = \sum_i \Big(\sum_k \alpha_{ik} N_{ik} z_{ik} + T_i\Big)/J.$

6. The compactness of each district must not exceed the allowable limit. Mathematically,

$$\sum_i \Big(\sum_k \alpha_{ik} N_{ik} z_{ik} + T_i\Big) d_{ij}^2 y_{ij} \le C \qquad \text{for all } j.$$

7. Each district formed must be contiguous. Mathematically,

For all $i \epsilon \{S_j\}$, if there exists an $i' \epsilon \{S_j\}$ such that $i \ne i$, then $i' \epsilon \{A_j\}$, for all j.

8. Each area is assigned to one and only one district. Mathematically,

$$\sum_j y_{ij} = 1, \quad \text{for all } i, \quad \text{and} \quad \sum_i y_{ij} \ge 1, \quad \text{for all } j.$$

Using the above mathematical formulation, the model of the decision problem is

$$\underset{\underline{z},\underline{y}}{\text{Minimize}} \sum_i \sum_k v_{ik} N_{ik} \sum_\ell \ell p_{ik\ell}(z_{ik}) \qquad \text{(P1)}$$

$$\text{S.T.:} \quad (1) \quad \sum_i \Big(\sum_k \alpha_{ik} N_{ik} z_{ik} + T_i\Big) \le M - h_4,$$

(2) $L_{ik} \leq z_{ik} \leq U_{ik}$ for all i,k,

(3) $z_{ik} = z_{i'k'}$, for all i-k and i'-k' such that

$$v_{ik} \sum_{\ell} \ell p_{ik\ell}(z_{ik}) = v_{i'k'} \sum_{\ell} \ell p_{i'k'\ell}(z_{i'k'})$$

and

$$L_{ik} = L_{i'k'} \text{ and } U_{ik} = U_{i'k'}.$$

(4) $Pz_{ik} \equiv 0 \pmod 1$ for all i,k,

(5) $(1-\Delta)\bar{T} \leq \sum_{i}\left(\sum_{k} \alpha_{ik}N_{ik}z_{ik}+T_i\right)y_{ij} \leq (1+\Delta)\bar{T}$ for all j,

(6) $\sum_{i} \left(\sum_{k} \alpha_{ik}N_{ik}z_{ik}+T_i\right)d_{ij}^2 y_{ij} \leq C$ for all j,

(7) For all $i\epsilon\{S_j\}$, if there exists an $i'\epsilon\{S_j\}$ such that $i \neq i'$, then $i'\epsilon\{A_i\}$ for all j,

(8) $\sum_{j} y_{ij} = 1$ for all i,

(9) $\sum_{i} y_{ij} \geq 1$ for all j,

(10) $y_{ij} = 1$ if area i assigned to district j; 0, otherwise, for all i,j.

In the above model, the general form for the "effectiveness function," $p_{ij\ell}(z)$, is used. However, before the model can be "solved," an explicit statement of p(z) must replace its general form in the model. Because p(z) is a stochastic function, an *estimator* of p(z) must be derived. A description of the procedure used in this research to obtain such an estimator, $\hat{p}(z)$, was presented in Chapter IV.

As mentioned in Chapter IV, the original estimator of p(z) is a composite of subjective and objective estimates, and is a nonincreasing function. However, it might be convex or concave in form. The approach used in this research is to approximate the original $\hat{p}(z)$ with a piece-wise linear function which is constrained to be convex in form. This function consists of two segments defined over the interval $[L_{ik},U_{ik}]$. The derivation of this approximate function was presented in Chapter IV.

Use of this piece-wise linear function, $\hat{p}_{ik\ell}(z)$, necessitates the replacement of z_{ik} by two auxiliary variables, z_{ik1} and z_{ik2}. See Hillier and Lieberman [35] for a discussion of the use of auxiliary variables and piece-wise linear approximation in general.

Model (P1) must be reformulated to include the explicit form of $\hat{p}_{ik\ell}(z)$ and to replace z_{ik} by z_{ik1} and z_{ik2}. In order to do this, make the following definitions.

Let

$$z_{ik1} = \begin{cases} z_{ik} & \text{if } z_{ik} \in [L_{ik},B_{ik}], \\ B_{ik} & \text{if } z_{ik} > B_{ik}, \end{cases}$$

$$z_{ik2} = \begin{cases} 0 & \text{if } z_{ik} \geq B_{ik}, \\ z_{ik}-B_{ik} & \text{if } z_{ik} \epsilon (B_{ik}, U_{ik}], \end{cases}$$

z = the matrix of all z_{ik1} and z_{ik2} values.

$s_{ik\ell 1}$ = the slope of the piece-wise function $\hat{p}_{ik\ell}(z)$ when $z_{ik} \epsilon [L_{ik}, B_{ik}]$,

$s_{ik\ell 2}$ = the slope of $\hat{p}_{ik\ell}(z)$ when $z_{ik\ell}(B_{ik}, U_{ik}]$.

(See Chapter IV for derivation of s.)

Note that the above definitions imply that

$$L_{ik} \leq z_{ik1} \leq B_{ik} \quad \text{and} \quad 0 \leq z_{ik2} \leq (U_{ik}-B_{ik}),$$

and

$$z_{ik} = z_{ik1} + z_{ik2}.$$

Also note that

$$\hat{p}_{ik\ell}(z_{ik}) = s_{ik\ell 1} z_{ik1} + s_{ik\ell 2} z_{ik2},$$

so that the criterion function in the model becomes

$$\sum_i \sum_k v_{ik} N_{ik} \sum_\ell \ell (s_{ik\ell 1} z_{ik1} + s_{ik2} z_{ik2})$$

or, equivalently,

$$\sum_i \sum_k (v_{ik} N_{ik} \sum_\ell \ell s_{ik\ell 1}) z_{ik1} + \sum_i \sum_k (v_{ik} N_{ik} \sum_\ell \ell s_{ik\ell 2}) z_{ik2}$$

Letting $c_{ik1} = v_{ik} N_{ik} \sum_\ell \ell s_{ik\ell 1}$, and $c_{ik2} = v_{ik} N_{ik} \sum_\ell \ell s_{ik\ell 2}$, the model (P1) can be rewritten as

$$\underset{\underset{\sim}{z},\underset{\sim}{y}}{\text{Minimize}} \sum_i \sum_k (c_{ik1} z_{ik1} + c_{ik2} z_{ik2}) \qquad \text{(P2)}$$

S.T.: (1) $\sum_i \left(\sum_k \alpha_{ik} N_{ik} z_{ik1} + \alpha_{ik} N_{ik} z_{ik2} + T_i\right) \le M - h_4$,

(2) $L_{ik} \le z_{ik1} \le B_{ik}$, for all i,k,

(3) $0 \le z_{ik2} \le (U_{ik} - B_{ik})$, for all i,k,

(4) $z_{ik1} = z_{i'k'1}$ and $z_{ik2} = z_{i'k'2}$
for all i-k and i'-k'

such that

$c_{ik1}/N_{ik} = c_{i'k'1}/N_{i'k'}$ and

$c_{ik2}/N_{ik} = c_{i'k'2}/N_{i'k'}$

and

$L_{ik} = L_{i'k'}$ and $U_{ik} = U_{i'k'}$

(5) $Pz_{ik1} = 0 \pmod 1$

and

$$Pz_{ik2} \equiv 0 \pmod 1), \qquad \text{for all } i,k,$$

$$(6)\quad (1-\Delta)\bar{T} \le \sum_i \Big(\sum_k \alpha_{ik} N_{ik}(z_{ik1}+z_{ik2})+T_i\Big) y_{ij}$$

$$\le (1+\Delta)\bar{T}, \qquad \text{for all } j$$

$$(7)\quad \sum_i \Big(\sum_k \alpha_{ik} N_{ik}(z_{ik1}+z_{ik2})\Big) T_i \, d_{ij}^2 y_{ij} \le C, \qquad \text{for all } j,$$

(8) For all $i \epsilon \{S_j\}$, if there exists an $i' \epsilon \{S_j\}$ such that $i \ne i'$, then $i' \epsilon \{A_i\}$, for all j,

$$(9)\quad \sum_j y_{ij} = 1 \qquad \text{for all } i,$$

$$(10)\quad \sum_i y_{ij} \ge 1 \qquad \text{for all } j,$$

(11) $y_{ij} = 1$ if area i is assigned to district j; 0, otherwise, for all i,j.

It should be pointed out that z_{ik1} and z_{ik2} must also be restricted so that z_{ik1} is at its upper bound before z_{ik2} is allowed to have a nonzero value. However, it is not necessary to add this constraint to model (P2) as it is satisfied automatically since $\hat{p}_{ik\ell}(z)$ is convex (see Hillier and Lieberman [35], p. 584).

Several observations should be made concerning the formulation of models (P1) and (P2). First, the formulations of the "work-time" functions, $T_{ik}^{(\cdot)}(z_{ik})$, assume that the parameters involved are independent of the solution to the overall problem. That is, the assumption is that the parameters are not affected by inspection frequencies or by the characteristics of districts. Actually, the frequency of scheduled routine inspections affects each of the following: (1) the time to conduct a routine inspection, α_{ik} (inspection time decreases as familiarity with the building and occupants increases), (2) the required number of reinspections, r_{ik} (the more routine inspections, the more reinspections), and (3) the number of fire reports to be investigated, b_i (inspection frequency affects the occurrence rate of fires).

Similarly, the composition of districts and the resulting characteristics of each district (compactness, etc.) affect (1) the time required for reinspections including travel time, θ_{ik} (travel time increases as compactness increases), and (2) the time required to investigate fire reports, γ_i (again, compactness can affect travel time).

As pointed out in Chapter III, very little is known at this time about the effects mentioned above. This lack of knowledge forces the assumption to be made that they are not significant. However, data to be collected in the continuation of the research project may allow these effects to be more accurately modelled.

Another assumption that has been incorporated into the model is that the acceptable limit on district workload deviation is independent of the acceptable limit on district compactness. Discussions with Bureau Management indicate that they have very little insight as to the impact of specific measurements of these two criteria. For example, a compactness measurement of 215 "work-hour miles" is practically meaningless to them. Likewise, a workload deviation measurement of 89 "work-hours" for the deviation of the workload of a district from the average over all districts has very little meaning by itself. Consequently, placing allowable limits on the absolute value of these measures is very difficult.

There is, nonetheless, a relationship between the measures of these criteria, and between the allowable limits on their measures. The relationships are due to trade-offs between the measures. For example, management may be more willing to accept a larger compactness measure if the workload deviation is decreased. However, with the current lack of insight into the impact of these criteria, the relationship between their acceptable limits cannot now be quantified. Therefore, the assumption must be made that they are independent.

Solution Procedure

The mathematical model (P2) presented in the previous section constitutes a mixed integer programming problem. In the application of this model to Atlanta data, there are 8024 variables each having a finite number of allowable fractional values (the z_{ik1} and z_{ik2} variables), 4130 0-1 integer variables (y_{ij}), and over 24,000 constraints. Fortunately, the problem has a special structure that can be taken advantage of in obtaining a solution.

Approach

Because of the nature of the constraints in model (P2), the problem can be decomposed into two subproblems. One subproblem consists of the objective function in problem (P2) and constraints (1)-(5). Note that this subproblem has the form of an integer "knapsack" problem with bounded variables. It should also be observed that this subproblem is independent of the districting decision variables, $\underline{y}$.

The second subproblem embedded in the overall model is the problem of finding a feasible set of districts. This subproblem results from constraints (6)-(11). Unfortunately, this districting subproblem is *not* independent of the inspection frequency decision variables, $\underline{z}$. But even if the two subproblems could be completely "decoupled," the size of each problem would still make an exact solution procedure appear impractical.

The approach used in this research was to develop a heuristic procedure to obtain a solution to the overall problem (P2). There is, of course, no guarantee that the global optimum will be obtained. Further, there is no guarantee that if a feasible solution exist it will necessarily be found (obtaining feasible solutions to this type of districting problem can be very difficult; see [22] for a discussion). Nonetheless, a heuristic procedure appears to be the most practical approach due to the size of the problem and the need for a relatively efficient algorithm.

The general scheme of the solution procedure is as follows:

1. obtain a "superoptimal" solution to (P2) by solving the

"knapsack" problem consisting of the objective function and constraints (1)-(5).

2. use this superoptimal solution, $\underline{z}^s$, to fix the district workloads, $\sum_i T_i(\underline{z}^s_{i.})y_{ij}$,

3. obtain a feasible solution to the districting subproblem consisting of constraints (6)-(11) and the fixed workloads,

4. if a feasible solution is found, stop; if not, hold the current "best" set of districts fixed and modify $\underline{z}^s$ until feasible districts are found; stop.

The procedure used to find a "superoptimal" solution to the "knapsack" subproblem is itself heuristic. It is a modification of the well-known procedure for solving the continuous version of the "knapsack" problem (or an "L.P." problem with a single constraint). The modification is three-fold. First, straightforward upper and lower bounding techniques areutilized to satisfy constraints (2) and (3). Second, a preliminary analysis prior to solution is performed to combine all those i-k categories meeting constraint (4). Those categories with the same parameters are treated in the solution procedure as a single decision variable by combining the objective function coefficients and the resource requirements of all such categories. Upon termination of the solution procedure, each group member is assigned the inspection frequency of the composite variable.

The third modification concerns "integerizing" the algorithm used to solve the continuous "knapsack" problem. Here is where the heuristic portion of the modified solution procedure comes into play. Specifically, once a variable has been chosen to enter the solution it is assigned as large a value as possible. Each of the remaining variables is examined to see if it can be assigned a value and thereby utilize any slack in constraint (1). This single pass may or may not result in all of the available man-hours being used.

The districting subproblem is very similar to the "political redistricting" problem described in Chapter II. The basic difference is that population is the unit of workload in the political redistricting problem whereas fire prevention man-hours is the unit of workload in the inspection districting problem. (Note that fire prevention man-hours is a *controllable* factor whereas population is an uncontrollable factor.)

Hess and Weaver [23] developed a heuristic procedure for a political redistricting problem having the same criteria as the inspection districting subproblem. A modification of their procedure is used in this research to obtain a set of feasible inspection districts. An outline of the basic steps in their procedure is as follows:

1. Guess initial district centers (centers of workload gravity).

2. Use a transportation algorithm to assign workload equally to these centers at minimum total squared distance of each area in a district to the center of that district.

3. Adjust assignments so that each area (census tract) is completely within one district.

4. Compute the centroid of each district and use as improved district centers.

5. Repeat from step (2) until solution converges.

6. Try more initial guesses of district centers.

While this method is not guaranteed to converge to a solution, Hess and Weaver report that experience with applications of the procedure provides no examples of nonconvergence. Further, they found that "so far all sets of guessed centers have converged to local minima in less than ten 'transportation' solutions" [23].

There is no provision in the Hess and Weaver method to ensure that when a solution is reached all of the districts will be contiguous. While their procedure does have a provision for recombining areas which are assigned to more than one district (i.e., "split" areas), there is no indication in their discussion of how this is to be done.

The procedure used to solve the inspection districting subproblem incorporates four modifications into the Hess and Weaver algorithm. First, a specific method is employed to reassign areas which are assigned to more than one district. The method attempts to reduce the workload imbalance among all districts by reassigning the split areas to the districts which have been assigned the largest portion of the area's workload. Because the split areas result from assigning workloads *equally* among all districts, this approach to reassigning split areas minimizes the workload deviation of the reassignments. In addition, reassignments are made only if contiguity requirements are met.

Second, checks are incorporated to prevent nonconvergence due to cycling. These checks are in the form of limits on the allowable number of iterations of various parts of the procedure.

The third modification is an operation which attempts to force noncontiguous solutions into contiguous solutions. This consists of penalizing the distance between the center of a district and each area causing that district to be noncontiguous. The districts are then re-defined using the penalized compactness measure.

The fourth modification is the addition of a procedure which attempts to force the redefining of districts which have workloads that are out of balance. It consists of a trial and error search of areas assigned to unbalanced districts in order to find areas which can be profitably reassigned to other districts.

<u>Algorithm</u>

Consider the following notation.

Let,

for m = 1,

$z_{ikm} = z_{ik1}$.

$c_{ikm} = c_{ik1}/N_{ik}$.

$L_{ikm} = L_{ik}$.

$U_{ikm} = B_{ik}$.

$\alpha_{ikm} = \alpha_{ik}$.

$\beta_{ikm} = \beta_{ik}$.

$r_{ikm} = r_{ik}$.

And for m = 2,

$z_{ikm} = z_{ik2}$.

$c_{ikm} = c_{ik2}/N_{ik}.$

$L_{ikm} = 0.$

$U_{ikm} = U_{ik} - B_{ik}.$

$\alpha_{ikm} = \alpha_{ik}.$

$\beta_{ikm} = 0.$

And

$r_{ikm} = 0.$

The steps inthe procedure to obtain a solution to the "Rate-District" problem as represented by model (P2) are included in the following outline.

Determine Superoptimal Rates

1. Eliminate constraint (3) by letting

$$c_{ikm} = \sum_{(i'-k')} c_{i'k'm}, \quad \text{for all } m,$$

$$\alpha_{ikm}N_{ik} = \sum_{(i'-k')} \alpha_{ikm}N_{ik}, \quad \text{for all } m,$$

$$r_{ikm} = \sum_{(i'-k')} r_{i'k'm}, \quad \text{for all } m,$$

and

$$\beta_{ikm}\alpha_{ikm}N_{ik} = \sum_{(i'-k')} \beta_{i'k'm}\alpha_{i'k'm}N_{i'k'}, \quad \text{for all } m$$

for all i-k and i'-k' combinations such that

$$c_{ik1} = c_{i'k'1}, \quad c_{ik2} = c_{i'k'2}, \quad L_{ik} = L_{i'k'},$$

and

$$U_{ik} = U_{i'k'}.$$

Remove these i'-k' combinations from the indices.

2. Set $z_{ikm} = L_{ikm}$ for all i,k,m (m=1,2). If constraint (1) is violated, stop; there is no feasible solution. If not, proceed.

3. Compute $E_{ikm} = (c_{ikm}N_{ik})/(\alpha_{ik}N_{ik})$, for all i,k,m.

4. Rank each i-k-m combination in descending order of magnitu de of E_{ikm}.

5. Compute $S = M - h_4 - \sum_i \left(\sum_k \sum_m \alpha_{ikm}N_{ikm}z_{ikm} + T_i\right)$.

6. If S = 0, let $\underline{z}^S = \underline{z}$ and go to step (9). If not, go to step (7). Repeat until the list of i-k-m combinations is empty.

7. For the first i-k-m combination in the ranked list compute

$$D = (S/\alpha_{ikm}N_{ik}) + L_{ikm}.$$

Round D down to its smallest value such that PD ≡ 0 (mod 1).

8. Let $z_{ikm} = \min(U_{ikm}, D)$ and remove i-k-m from the list. Go to step (5).

Determine Initial District Centers in Two Different Ways

9. Compute the workload for each area, w_i, as follows:

$$w_i = \sum_k \sum_m \alpha_{ikm} N_{ik} z^s_{ikm} + T_i.$$

Method One

10. Rank the areas in descending order of magnitude of w_i.

11. Let the initial district centers be the centers of gravity of the first J areas in the list. That is, define

x^d_j = the x-coordinate of the center of the *jth* district.

y^d_j = the y-coordinate of the center of the *jth* district.

x^a_i = the x-coordinate of the center of workload gravity of area i.

y^a_i = the y-coordinate of the center of workload gravity of area i.

And let

$$x^d_j = x^a_i \quad \text{and} \quad y^d_j = y^a_i \quad \text{for } j = 1,2,\ldots,J \text{ and } i = \text{first } J \text{ members of the list.}$$

Go to step (13).

Method Two

12. Guess the coordinates of J district centers.

13. Calculate distances between each area and each district as follows:

$$d_{ij} = \left[(x^d_j - x^a_i)^2 + (y^d_j - y^a_i)^2\right]^{1/2}$$

and

calculate d^2_{ij}.

Solve Transportation

14. Using (a) district centers as "sources" with equal capacities $\sum_i w_i/J$, (b) areas as "sinks" with demand requirements w_i, (c) $w_i d_{ij}^2$ as unit "costs," solve a J by I transportation problem with equality capacity constraints.

Recombine Split Areas

15. Assign each split area to the district in which it has its largest workload portion, provided the area is adjacent to some area in the district. Break ties arbitrarily. If no contiguous assignment is possible, make an arbitrary assignment.

Check Contiguity

16. For each district, determine whether or not all areas assigned to thatdistrict are adjacent to another area in the district. If all districts are contiguous, go to step (19). If not, proceed.

17. For each district, let

$d_{ij}^2 \to \infty$ for all i such that i is a nonadjacent member of district j (i.e., $i \epsilon \{S_j\}$ and $i' \notin \{A_i\}$ $\forall i' \epsilon \{S_i\}$ and $i' \neq i$).

18. If noncontiguous districts have been formed in the most recent BADMAX iterations of steps (14)-(17), go to step (28). If not, go to step (14).

Improve Workload Balance

19. If all districts are balanced, i.e., if for all j

$$(1-\Delta)\bar{T} \leq \sum_{i \epsilon \{S_j\}} w_i \leq (1+\Delta)\bar{T}$$

where $\bar{T} = \sum_i w_i/J$,

go to step (24). If not, repeat steps (20)-(22) for each district $j = j'$ that is out of balance.

20. For all j, let $W_j = \sum_{i\epsilon\{S_j\}} w_i$.

If j' is "underbalanced," i.e., $W_{j'} < (1-\Delta)\bar{T}$, go to step (22).

If j' is "overbalanced,"i.e., $W_{j'} > (1+\Delta)\bar{T}$, proceed.

21. Find an $i = i'$, such that $i'\epsilon\{S_{j'}\}$ and

(a) the set of areas $\{S_{j'}\} - i'$ is contiguous,

(b) $(1-\Delta)\bar{T} \leq W_{j'} - w_{i'} \leq (1+\Delta)\bar{T}$, and

(c) there exist a $j = j''$ such that $j'' \neq j'$, $\{S_{j''}\} + i'$ is contiguous, and $(1-\Delta)\bar{T} \leq W_{j''} + w_{i'} \leq (1+\Delta)\bar{T}$.

If such an i' exist,reassign i' to district j''. That is, let $\{S_{j'}\} \leftarrow \{S_{j'}\} - i'$, $\{S_{j''}\} \leftarrow \{S_{j''}\} + i'$, $W_{j'} \leftarrow W_{j'} - w_{i'}$, and $W_{j''} \leftarrow W_{j''} + w_{i'}$. Go to step (23).

22. Find an $i = i'$, such that $i'\epsilon\{S_{j''}\}$ for some $j = j''$, and $j'' \neq j'$, and

(a) the set of areas $\{S_{j'}\} + i'$ is contiguous,

(b) $\{S_{j''}\} - i'$ is contiguous.

(c) $(1-\Delta)\bar{T} \leq W_{j'} + w_{i'} \leq (1+\Delta)\bar{T}$, and

(d) $(1-\Delta)\bar{T} \leq W_{j''} - w_{i'} \leq (1+\Delta)\bar{T}$.

If such an i' exist,reassign i' to district j' and go to step (23). If not, attempt to find a "second level" of reassignments; that is, find an $i = i''$ such that $i''\epsilon\{S_{j*}\}$ for $j* \neq j'$ and $j* \neq j''$ and

(e) $\{S_{j*}\} - i''$ is contiguous,

(f) $\{S_{j''}\} + i''$ is contiguous,

(g) $(1-\Delta)\bar{T} \le W_{j''} + w_{i''} \le (1+\Delta)\bar{T}$,

(h) $(1-\Delta)\bar{T} \le W_{j*} - w_{i''} \le (1+\Delta)\bar{T}$, and

(i) there exist an i' as described in steps (22.a)-(22.d) for $i' \neq i''$, and $\{S_{j''}\} \leftarrow \{S_{j''}\} + i''$.

If such an i" exist, reassign i" to district j" and i' to district j'.

23. If all districts are balanced, go to step (24). If not, go to step (25).

24. If all districts are compact, i.e., if, for all j,

$$\sum_i w_i d_{ij}^2 \le C$$

go to step (28). If not, proceed.

25. If the most recent MAXBAL district solutions (resulting from steps (14)-(22)) have not been balanced or compact, go to step (28). If this is not true, proceed.

Complete Districting Procedure

26. If the current solution, i.e., the set of $\{S_j\}$ for all j, is the same as the previous solution go to step (28). If not, proceed.

27. Recompute district centers and distances as follows:

$$x_j^d = \Big(\sum_{i\epsilon\{S_j\}} w_i x_i^a\Big)/T, \quad \text{and} \quad y_j^d = \Big(\sum_{i\epsilon\{S_j\}} w_i y_i^a\Big)/T \quad \text{for all } j,$$

$$\text{where } T = \sum_i w_i,$$

and

$$d_{ij} = \big((x_j^d - x_i^a)^2 + (y_j^d - y_i^a)^2\big)^{1/2}.$$

Go to step (14).

28. If the current solution is based on the initial district centers of method one (i.e., steps (10)-(11)), go to step (29). If not, go to step (30).

29. Compute the "value," $\underset{\sim}{V} = (V_1,V_2,V_3,V_4)$, of the current solution as follows:

$$V_1 = \sum_j (W_j - \bar{T})^2$$

$$V_2 = 0 \text{ if } \sum_{i\epsilon\{S_j\}} w_i d_{ij}^2 \leq C \text{ for all } j;\ 1, \text{ otherwise.}$$

$$V_3 = 0 \text{ if } (1-\Delta)\bar{T} \leq W_j \leq (1+\Delta)\bar{T} \text{ for all } j;\ 1, \text{ otherwise.}$$

$$V_4 = 0 \text{ if all } j \text{ are contiguous; } 1, \text{ otherwise.}$$

Go to step (12).

30. Choose the best solution between the set of districts resulting from the two initial solutions as follows:

Let $\underset{\sim}{V}^{(1)}$ be the specific $\underset{\sim}{V}$ for the solution associated with the first initial district centers and $\underset{\sim}{V}^{(2)}$ be the specific $\underset{\sim}{V}$ for the solution associated with the second initial district centers.

Choose the first solution if

(a) $V_1^{(1)} \leq V_1^{(2)}$, $V_2^{(1)} = 0$, $V_3^{(1)} = 0$, and $V_4^{(1)} = 0$; or

(b) $V_1^{(1)} > V_1^{(2)}$, $V_2^{(1)} = 0$, $V_3^{(1)} = 0$, $V_4^{(1)} = 0$, and $V_2^{(2)} = 1$ or $V_3^{(2)} = 1$ or $V_4^{(2)} = 1$; or

(c) $V_1^{(1)} \leq V_1^{(2)}$, $V_2^{(1)} = 1$ or $V_3^{(1)} = 1$ or $V_4^{(1)} = 1$, and $V_2^{(2)} = 1$ or $V_3^{(2)} = 1$ or $V_4^{(2)} = 1$.

Otherwise, choose the second solution

31. If the solution has a "value" with component $V_3 = 0$, stop. If not, proceed.

Modify Inspection Rates

32. For those districts, j', that are overbalanced (i.e., $W_{j'} > (1+\Delta)\bar{T}$) reduce $W_{j'}$ as follows:

(a) For all $i \epsilon \{S_{j'}\}$, rank all i-k-m combinations in increasing order of magnitude of

$$E_{ikm} = (c_{ikm}N_{ik})/(\alpha_{ikm}N_{ik}).$$

(b) For the first i-k-m in the list compute

$$D_1 = \left(W_{j'}-(1+\Delta)\bar{T}\right)/(\alpha_{ikm}N_{ik})$$

and

$$D_2 = z_{ikm} - L_{ikm}.$$

(c) Let $z_{ikm} \leftarrow z_{ikm} - \min(D_1,D_2)$. Round z_{ikm} down until

$Pz_{ikm} \equiv 0 \pmod 1$.

Let $W_{j'} \leftarrow W_{j'} - \alpha_{ikm}N_{ik}\left(\min(D_1,D_2)\right)$.

Remove this i-k-m from the list.

(d) If $(1-\Delta)\bar{T} \leq W_{j'} \leq (1-\Lambda)\bar{T}$, stop. If not, go to step (32.b).

For those districts, j', that are underbalanced $\left(\text{i.e., } W_{j'} < (1-\Delta)\bar{T}\right)$ increase $W_{j'}$ as follows:

(e) For all $i \in \{S_{j'}\}$, rank all i-k-m combinations in decreasing order of magnitude of

$$E_{ikm} = (c_{ikm}N_{ik})/(\alpha_{ikm}N_{ik}).$$

(f) For the first i-k-m in the list compute

$$D_1 = \left((1-\Delta)\bar{T}-W_{j'}\right)/(\alpha_{ikm}N_{ik})$$

and

$$D_2 = U_{ikm} - z_{ikm}.$$

(g) Let $z_{ikm} \leftarrow z_{ikm} + \min(D_1,D_2)$. Round z_{ikm} down until $pz_{ikm} \equiv 0 \pmod 1$. Let $W_{j'} \leftarrow W_{j'} + \alpha_{ikm}N_{ik}\left(\min(D_1,D_2)\right)$. Remove this i-k-m from the list.

(h) If $(1-\Delta)\bar{T} \leq W_{j'} \leq (1+\Delta)\bar{T}$, stop. If not, go to step (32.f).

A flow diagram of the districting portion of this algorithm is given in Figure 5. Note that the input to this part of the algorithm is a set of superoptimal inspection frequencies found in the "knapsack" subproblem.

Several observations should be made concerning the solution procedure. First, only two sets of initial district centers are used as starting points for the districting subproblem. Obviously, additional initial sets could be used. Second, the algorithm used in this research to solve the transportation subproblem is the "MODI" method of Hadley [36]. It is a primal algorithm and has been coded in Fortran by IBM [37].

Another point that should be made concerns the preference for one districting solution over another. Such a choice must be made between the solutions resulting from the use of each set of initial district centers. If one solution "dominates" the other, then it should be chosen. Domination implies that either (1) all three constraints of the districting criteria (compactness, workload balance, and contiguity) are met for both solutions but one solution has a smaller sum of the squares of workload deviation, or (2) all three criterion constraints are met for one solution but not for the other.

If there is no dominant districting solution, the one with the smaller sum of the squares of workload deviation is chosen. This rule is based on the Bureau's preference for workload balance as the main criteria for districting (see step (30) in the algorithm).

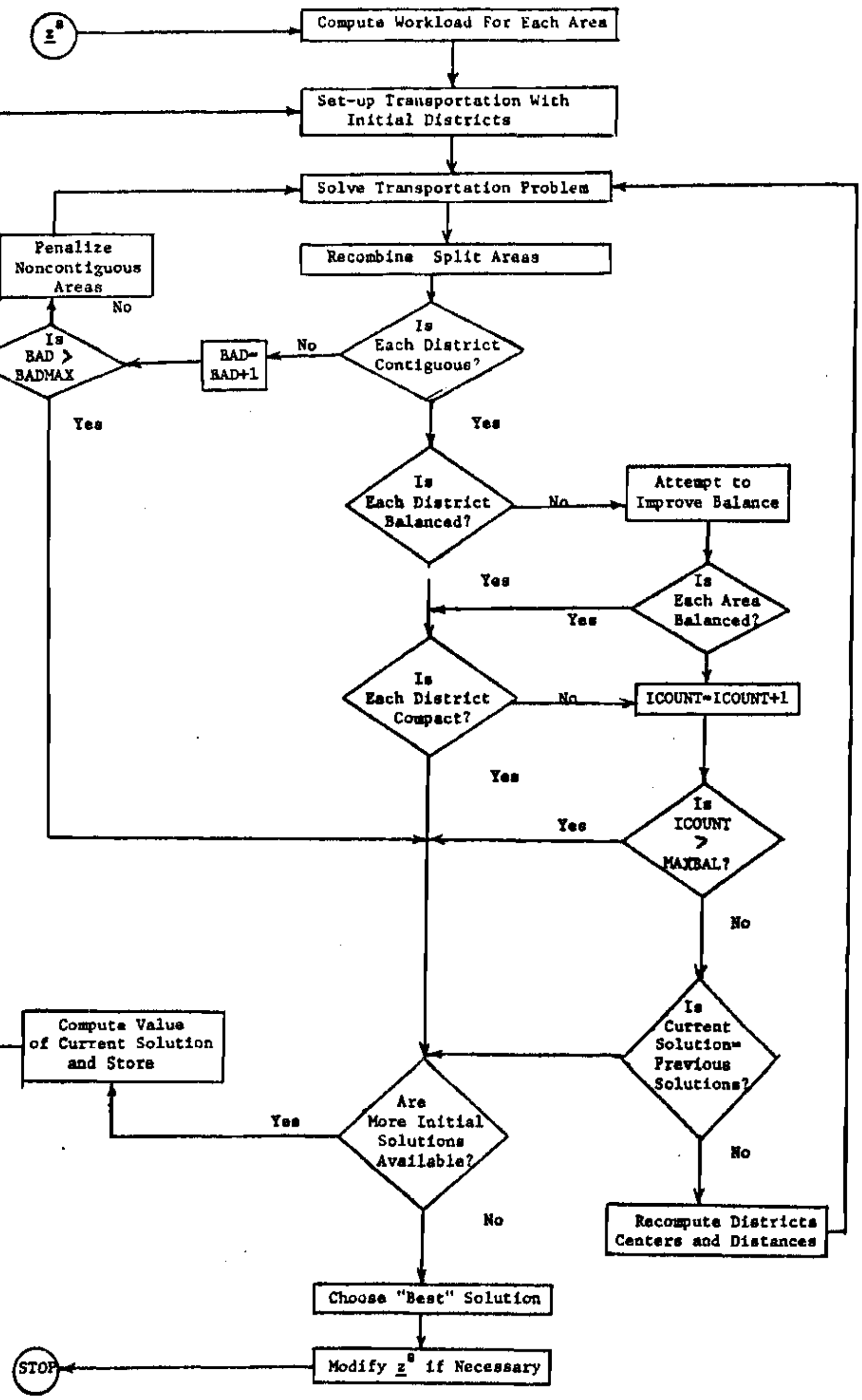


Figure 5. Flow Diagram of Districting Procedure

The 32-step solution procedure was coded in Fortran IV to be run on a Univac 1108 system. An example problem was constructed to test the computer code and to provide insight into the validity of the solution procedure. The hypothetical example consisted of 2 occupancy types (K = 2) and 8 census tracts (I = 8). The number of districts required (J) was varied between 2 and 5. The number of structures in each area (N_{ik}) was also varied in different "runs." Other parameter values were set in such a manner as to test the ability of the algorithm to handle extreme applications. The solution procedure worked effectively and was efficient on this relatively small size example problem. The results of the application of the solution procedure to the Atlanta data are given in Chapter VII.

CHAPTER VI

DETERMINATION OF A SCHEDULE FOR ROUTINE INSPECTIONS

The model presented in Chapter V can be used by Bureau Management to determine how often to inspect various categories of buildings, and to determine inspection districts. Once these two decisions have been made, the date of each inspection in every building in all districts must be determined. The criterion and constraints involved in this decision were discussed in Chapter III. In this chapter, the formulation of a mathematical model of the problem will be presented. Features of the model which are relevant to the design of a solution procedure will be described. However, no attempt has been made to develop a specific algorithm. Discussion will also be given of the updating or revision of the schedule which must take place each week.

Model

As mentioned in Chapter III, the scheduling decision in a district is assumed to be independent of scheduling considerations in other districts. This allows the overall scheduling problem to be decomposed into separate subproblems, one for each of the $j = 1,2,\ldots,J$ districts.

The criterion of each problem is the balance of the workload of the inspector from week to week. The measure defined in Chapter III for this criterion is the sum of the squared deviations of the workload each week from the average weekly workload. It should be noted that the scheduling decision affects only part of the weekly workload--that associated with assigned routine inspections. The remainder of the workload results from the weekly "demand" for the other four fire prevention operations.

There are four restrictions on allowable schedules for a district. These are (1) "frequency satisfaction," (2) "weekly workload feasibility," (3) "timing feasibility," and (4) "neighbor inspection feasibility." A description of each constraint was given in Chapter III.

Notation

Define the following terms:

Decision Variables

$x_{i\ell t}$ = 1 if the ℓth building in area (census tract) i is inspected in the $t th$ week of the P-year planning period ($\ell = 1,2,\ldots,B_i$; $i = 1,2,\ldots,L$; and $t = 1,2,\ldots,H$).

$\underline{x}_{\cdot\cdot t}$ = the vector $(x_{11t}, x_{12t}, \ldots, x_{IB_i t})$.

$\underline{x}_{i\ell\cdot}$ = the vector $(x_{i\ell 1}, x_{i\ell 2}, \ldots, x_{i\ell H})$.

$\underline{x}$ = the vector $(\underline{x}_{\cdot\cdot 1}, \underline{x}_{\cdot\cdot 2}, \ldots, \underline{x}_{\cdot\cdot H})$.

$\{X_t\}$ = the set of all $\underline{x}_{\cdot\cdot t}$ which meet the "neighbor inspection feasibility" constraint for week t.

$\{X_{i\ell}\}$ = the set of all $\underline{x}_{i\ell\cdot}$ which meet the "timing feasibility" constraint for building ℓ in area i.

Parameters

$z_{i\ell}$ = the number of scheduled, routine inspections to be conducted in the ℓth building in area i ($z_{i\ell} = z_{ik}$ for k such that the ℓth building is a type k occupancy).

$\alpha'_{i\ell t}$ = the time required to conduct a routine inspection in the ℓth building in area i in week t.

β_{ikt} = the percentage of N_{ik} structures requiring nonscheduled, routine inspections in week t.

r_{ikt} = the required number of reinspections of occupancy type k structures in area i in week t.

b_{it} = the required number of fire report investigations in area i in week t.

h_{ik1t} = the number of i-k nonstructural inspections required in week t.

h_{ik2t} = the number of i-k complaints answered in week t.

h_{i3t} = the number of "new building" inspections required in area i in week t.

h_{4t} = the remaining indirect work time required in week t.

Functions

$T_{jt}(\underline{x}_{\cdot\cdot t})$ = the workload of fire prevention man-hours required in district j in week t, given $\underline{x}_{\cdot\cdot t}$.

$$T_{jt}(\underline{x}_{\cdot\cdot t}) = \sum_{i\epsilon\{S_j\}} \Big(\sum_{\ell} \alpha'_{i\ell t} x_{i\ell t} + \sum_k \big(\beta_{ikt}\alpha_{ik}N_{ik} + \theta_{ik}r_{ikt} + \phi_{ik1}h_{ik1t} + \phi_{ik2}h_{ik2t}\big) + \phi_{i3}h_{i3t} + \gamma_i b_{it}\Big) + h_{4t}$$

$\bar{T}_j(\underline{x}) = \sum_t T_{jt}(\underline{x}_{\cdot\cdot t})/H.$

C_{jt} = the man-hours available for fire prevention operations in district j in week t.

The notation of Chapter V is applicable to this model and remains as defined there unless changed above. In order that the annual workload for each inspector be entirely accounted for, the following relation must hold (since W_j is an annual figure)

$$PW_j = \sum_t T_{jt}(\underline{x}_{\cdot\cdot t}), \quad \text{for all } j,$$

which implies that the above parameters be defined such that

$$\sum_{} \sum_{\ell} \alpha'_{i\ell t} x_{i\ell t} = P\alpha_{ik} N_{ik} z_{ik},$$

$$\sum_{t} \beta_{ikt} \alpha_{ik} N_{ik} = P\beta_{ik} \alpha_{ik} N_{ik},$$

$$\sum_{t} \theta_{ik} r_{ikt} = P\theta_{ik} r_{ik},$$

$$\sum_{t} \gamma_i b_{it} = P\gamma_i b_i,$$

$$\sum_{t} \phi_{ik1} h_{ik1t} = P\phi_{ik1} h_{ik1},$$

$$\sum_{t} \phi_{ik2} h_{ik2t} = P\phi_{ik2} h_{ik2},$$

$$\sum_{t} \phi_{i3} h_{i3t} = P\phi_{i3} h_{i3},$$

$$\sum_{t} h_{4t} = Ph_4,$$

and

$$\sum_{j} \sum_{t} C_{jt} = PM.$$

Mathematical Model

The criterion for the scheduling problem is the sum of the squared deviation of the weekly workload from the average weekly workload. Therefore, the objective function can be written as

$$\underset{\underline{x}}{\text{Minimize}} \sum_{t} \left(T_{jt}(\underline{x}_{..t}) - \bar{T}_j(\underline{x})\right)^2,$$

or, equivalently,

$$\underset{\underline{x}}{\text{Minimize}} \sum_t \Big(\sum_{i\epsilon\{S_j\}} \big(\sum_\ell \alpha'_{i\ell t} x_{i\ell t} + \sum_k (\beta_{ikt}\alpha_{ik}N_{ik}$$

$$+\theta_{ik}r_{ikt}+\phi_{ik1}h_{ik1t}+\phi_{ik2}h_{ik2t})$$

$$+ \phi_{i3}h_{i3t} + \gamma_i b_{it}\big) + h_{4t} - \bar{T}_j(\underline{x})\Big)^2,$$

For convenience, let

$$T'_{jt} = \sum_{i\epsilon\{S_j\}} \big(\sum_k \beta_{ikt}\alpha_{ik}N_{ik}+\theta_{ik}r_{ikt}+\phi_{ik1}h_{ik1t}+\phi_{ik2}h_{ik2t}) + \phi_{i3}h_{i3t} + \gamma_i b_{it}\big)$$

The mathematical model representing the scheduling decision problem for the j*th* district is as follows (the constraint numbering is consistent with that of the Chapter III discussion):

$$\underset{\underline{x}}{\text{Minimize}} \sum_t \Big(\sum_{i\epsilon\{S_j\}} \sum_\ell \alpha'_{i\ell t}x_{i\ell t}+T'_{jt}-\bar{T}_j(\underline{x})\Big)^2 \qquad \text{(P3)}$$

Subject To:

(1) $\sum_t x_{i\ell t} = z_{i\ell}$ for all i,ℓ,

(2) $\sum_{i\epsilon\{S_j\}} \sum_\ell \alpha'_{i\ell t}x_{i\ell t} + T'_{jt} \leq C_{jt}$ for all t,

(3) $\underline{x}_{i\ell\cdot}\epsilon\{X_{i\ell}\}$ for all i,ℓ,

(4) $\underline{x}_{\cdot\cdot t} \in \{X_t\}$ for all t,

(5) $x_{i\ell t} = 1$, or 0 for all i,ℓ,t.

Solution Approach

This model has several features which should influence the design of a solution procedure. Some of these are included in the following list.

1. *Size.* In the Atlanta application, there are 118 census tracts in the city (I = 118), an average of 150 buildings requiring routine inspections in each tract ($B_i \cong 150$), 104 weeks in the planning horizon, and an average of 6 tracts assigned to each district. Therefore, the above model has 93,600 0-1 decision variables ($x_{i\ell t}$) and 2008 primary constraints. Even though the constraint set has a special structure, this large number of 0-1 variables makes an exact solution approach appear impractical.

2. *"Transportation" Constraints.* Constraints (1)-(2) resemble the source capacity and sink requirements constraints of a transportation problem. This is apparent when constraint (2) is rewritten as

$$\sum_{i\in\{S_j\}} \sum_{\ell} \alpha'_{i\ell t} x_{i\ell t} \le C'_{jt} \quad \text{for all } t,$$

where

$$C'_{jt} = C_{jt} - T'_{jt},$$

and the right-hand side of the inequality is constant for a given j and t.

The difference between these two constraints and the transportation constraints is the $\alpha'_{i\ell t}$ coefficient in constraint (2). Although a standard transportation algorithm can not be applied as a subprocedure, this basis structure may be useful in a solution procedure. For example, schemes used to form and modify cell allocations in a transportation matrix might be adapted to account for $\alpha'_{i\ell t}$. This could be useful in obtaining a feasible solution to problem (P3). The matrix structure for such allocations is given in Figure 6.

3. *Structure of Feasible Sets.* The criteria used to define the two sets $\{X_{i\ell}\}$ and $\{X_t\}$ can affect the design of a solution procedure. If there are no restrictions on either the "timing dimension" ($\underline{x}_{i\ell\cdot}$) or the "neighbor proximity dimension" ($\underline{x}_{\cdot\cdot t}$) of a schedule, then $\{X_{i\ell}\}$ and $\{X_t\}$ are complete. That is, all possible schedules are included in these sets. In this case, constraints (3) and (4) can be removed from the problem.

A more realistic situation is that management will impose specific criteria to restrict feasible schedules. Some criteria for defining both $\{X_{i\ell}\}$ and $\{X_t\}$ were described in Chapter III. The method of generating feasible schedules depends on which criteria were used. For example, the method used if the date of inspections and proximity to neighbor inspections are to be random would be different than the method used if the time between inspections must be uniform.

4. *Lower Bound on Objective Function.* The nonlinear objective

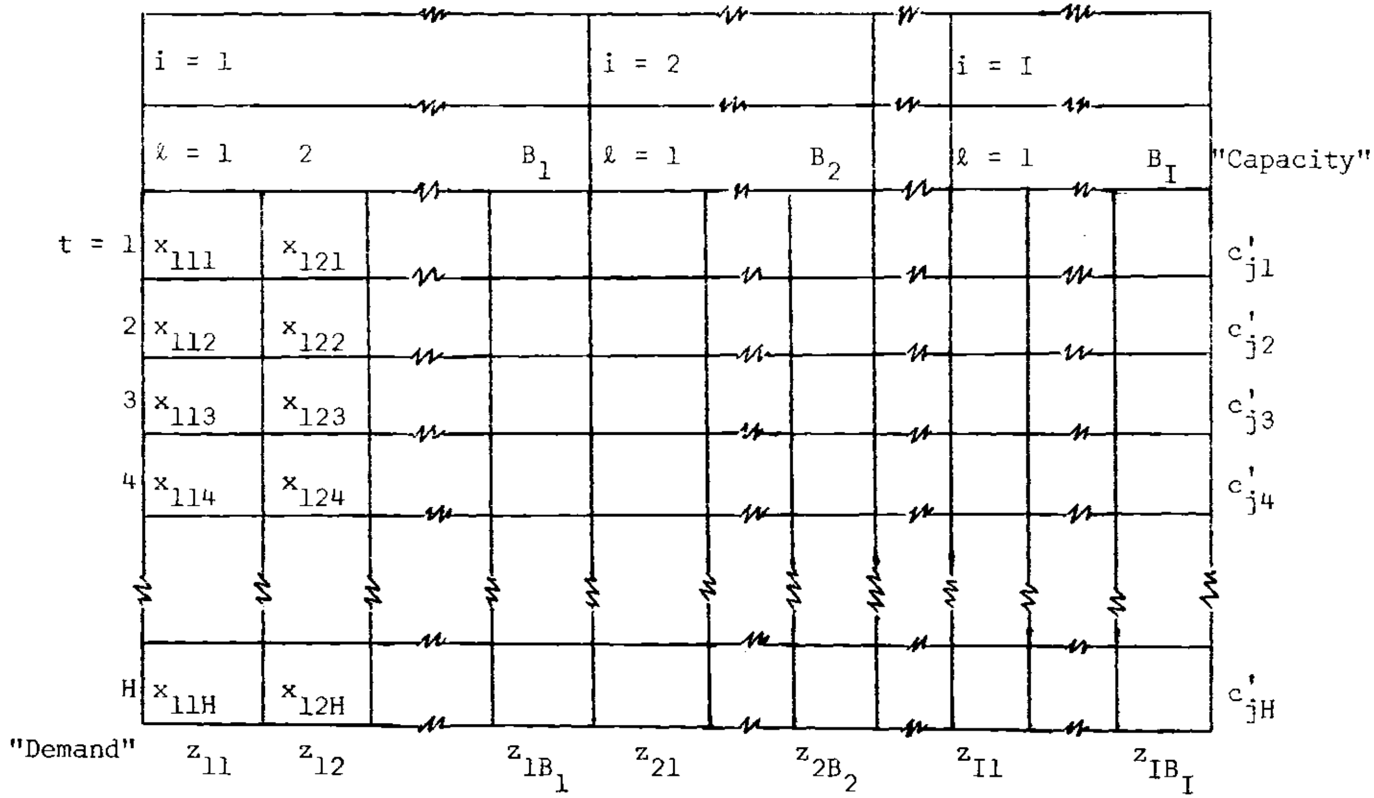


Figure 6. Allocation Matrix for Scheduling Problem

function of problem (P3) can be rewritten as

$$\underset{\underline{x}}{\text{Minimize}} \sum_t \Big(\sum_{i \in \{S_j\}} \sum_\ell \alpha'_{i\ell t} x_{i\ell t} - \bar{T}'_{jt} \Big)^2$$

where

$$\bar{T}'_{jt} = PW_j/H - T'_{jt}.$$

After completingthe square, this can be rewritten as

$$\underset{\underline{x}}{\text{Minimize}} \sum_t \left\{ \Big(\sum_{i \in \{S_j\}} \sum_\ell \alpha_{i\ell t} x_{i\ell t} \Big)^2 - 2 \sum_{i \in \{S_j\}} \sum_\ell \alpha'_{i\ell t} x_{i\ell t} \bar{T}'_{jt} + (\bar{T}'_{jt})^2 \right\}$$

Let

$$(\text{Eq.1}) \equiv \sum_t \left[\Big(\sum_{i \in \{S_j\}} \sum_\ell \alpha'_{i\ell t} x_{i\ell t} \Big)^2 - 2 \sum_{i \in \{S_j\}} \sum_\ell \alpha'_{i\ell t} x_{i\ell t} \bar{T}'_j(\underline{x}) + (\bar{T}'_{jt})^2 \right]$$

and

$$(\text{Eq.2}) \equiv \sum_t \left\{ \sum_{i \in \{S_j\}} \sum_\ell (\alpha'_{i\ell t})^2 x^2_{i\ell t} - 2 \sum_{i \in \{S_j\}} \sum_\ell \alpha'_{i\ell t} x_{i\ell t} \bar{T}'_j(\underline{x}) + (\bar{T}'_{jt})^2 \right\}$$

Since it is well known that the sum of the squares of nonnegative terms is less than or equal to the square of the sum of those terms, it follows that

$$(\text{Eq. 2}) \le (\text{Eq. 1})$$

for each $\underline{x}$. Since this holds for any $\underline{x}$, it holds for all $\underline{x}$ so that

$$\underset{\underline{x}}{\text{Minimize}}\ (\text{Eq. } 2) \leq \underset{\underline{x}}{\text{Minimize}}\ (\text{Eq. } 1).$$

Since $x_{i\ell t} = 1$ or 0 for all i, ℓ, t, then $x_{i\ell t}^2 = 1$ or 0, and $x_{i\ell t}^2 = x_{i\ell t}$, so that

$$(\text{Eq.}2) = \sum_t \left\{ \sum_{i\epsilon\{S_j\}} \sum_\ell (\alpha'_{i\ell t})^2 x_{i\ell t} - 2 \sum_{i\epsilon\{S_j\}} \sum_\ell \alpha'_{i\ell t} x_{i\ell t} \bar{T}'_{jt} + (\bar{T}'_{jt})^2 \right\},$$

or upon rearranging terms and bringing the sum over t inside the parenthesis,

$$(\text{Eq.}2) = \sum_{i\epsilon\{S_j\}} \sum_\ell \sum_t \left((\alpha'_{i\ell t})^2 - 2\bar{T}'_{jt}\alpha_{i\ell t}\right) x_{i\ell t} + \sum_t (\bar{T}'_{jt})^2.$$

Letting $c_{i\ell t} = (\alpha'_{i\ell t})^2 - 2\bar{T}'_{jt}\alpha_{i\ell t}$,

$$(\text{Eq. } 2) = \sum_{i\epsilon\{S_j\}} \sum_\ell \sum_t c_{i\ell t} x_{i\ell t} + \sum_t (\bar{T}'_{jt})^2.$$

Therefore, the lower bound on the objective function of problem (P3) is found by solving the following problem:

$$\underset{\underline{x}}{\text{Minimize}} \sum_{i\epsilon\{S_j\}} \sum_\ell \sum_t c_{i\ell t} x_{i\ell t} + \sum_t (\bar{T}'_{jt})^2$$

S.T.: Constraints (1)-(5) in (P3).

An alternative lower bound exists if it is assumed that $\alpha'_{i\ell t} = \alpha'_{i\ell}$ for all t. This implies that the time to conduct a routine inspection is independent of the week in which it is conducted. Rewriting (Eq. 1) and (Eq. 2),

$$\text{(Eq. 1)} = \sum_t \left[\left(\sum_{i\epsilon\{S_j\}} \sum_\ell \alpha'_{i\ell} x_{i\ell t}\right)^2 - 2 \sum_{i\epsilon\{S_j\}} \sum_\ell \alpha'_{i\ell} x_{i\ell} \bar{T}'_{jt} + (\bar{T}'_{jt})^2\right],$$

and

$$\text{(Eq. 2)} = \sum_{i\epsilon\{S_j\}} \sum_\ell (\alpha'_{i\ell})^2 \sum_t x^2_{i\ell t} - 2 \sum_{i\epsilon\{S_j\}} \sum_\ell \alpha'_{i\ell} \sum_t x_{i\ell t} \bar{T}'_{jt} + \sum_t (\bar{T}'_{jt})^2.$$

Since $x^2_{i\ell t} = x_{i\ell t}$, and since a feasible solution to (P3) has $\sum_t x_{i\ell t} = z_{i\ell}$, then

$$\text{(Eq. 3)} \equiv \sum_{i\epsilon\{S_j\}} \sum (\alpha'_{i\ell})^2 z_{i\ell} - 2 \sum_{i\epsilon\{S_j\}} \sum_\ell \alpha'_{i\ell} \sum_t \bar{T}'_{jt} x_{i\ell t} + \sum_t (\bar{T}'_{jt})^2$$

Since (Eq. 3) = (Eq. 2) $\leq$ (Eq. 1) for all $\underline{x}$, then a lower bound on (P3) can be found by solving the following problem:

$$\underset{\underline{x}}{\text{Minimize}} \; - 2 \sum_{i\epsilon\{S_j\}} \sum_\ell \sum_t \alpha'_{i\ell} \bar{T}'_{jt} x_{i\ell t} + A$$

S.T.: Constraints (1)-(5) of Problem (P1)

where $A = \sum_{i\epsilon\{S_j\}} \sum_\ell (\alpha'_{i\ell})^2 z_{i\ell} + \sum_t (\bar{T}'_{jt})^2$.

No attempt to develop a specific solution procedure for problem (P3) is made in this research. However, the model features mentioned above should be useful in the future design of such a procedure. For example, it appears that a heuristic procedure must be used in view of the size of potential applications. Also, a primal approach appears practical in which a feasible solution is first found by using a modified transportation procedure, then improved as much as possible.

Weekly Updating

The scheduling problem discussed above must be resolved for each inspection district at the beginning of each two-year planning period. Weekly assignments of routine inspections will be given each inspector based on this schedule. However, it is unlikely that this planned schedule can be followed. The reason is that most of the parameters in the scheduling model have *probabilistic* values. The quantities used in solving the problems are *estimates* of parameter values made at the beginning of the planning period. Obviously, these estimates are not exact and can differ from the actual values of the parameters realized *during* the planning period. This can cause a "good" planned schedule to become infeasible.

For example, the number of fire reports requiring investigation in each area in each week, R_{it}, is needed as input to the scheduling model. Since b_{it} is a random variable, an estimate must be made at the beginning of the two-year planning horizon of b_{it}. Suppose that $\hat{b}_{12}$ = 10 (reports) is used. It may happen that 30 reports require investigation in area 1 in week, i.e., b_{12} = 30. The additional 20 reports consume time that was planned for use by other operations, including scheduled, routine inspections.

In Atlanta, practice dictates that a lower priority be placed on scheduled, routine inspections than on the other four fire prevention operations. That is, when a "demand" for these other operations occurs they are attended to while scheduled, routine inspections are allowed to "slip." This priority scheme is not based on an evaluation of the relative value of each operation but rather on pragmatic considerations.

When unpredicted circumstances arise and scheduled, routine inspections are not conducted in the week they are assigned, the schedule for the remaining portion of the planning horizon must be updated or revised. Otherwise, time will be spent in inspecting low-value buildings (e.g., service stations) that could more profitably be spent in conducting inspections in high-value buildings (e.g., hospitals) that were missed. In other words, all buildings in a district are competing for available inspection time in each week in the remaining portion of the planning period.

The decision of how to update the remaining schedule for a district at the end of any week constitutes a "Rate Determination" problem as discussed in Chapter III. In this case, the "Rate Determination" problem is independent of the "Inspection District" problem. The relevant planning horizon is the remaining number of weeks left in the original two-year planning period. No attempt will be made here to develop a model or solution procedure for this updating problem.

CHAPTER VII

APPLICATION

The model developed in Chapter V was used to analyze the problem of setting inspection rates and determining inspection districts for the Atlanta Fire Department. This involved obtaining numerical estimates of all the parameters in the model, and using the coded algorithm described in Chapter V to obtain a solution. This chapter contains a description of the procedures used to obtain parameter estimates and the data sources which were involved. Also, a discussion is given of the solution found.

Estimation of Parameters

There are two basic types of parameters in model (P2)--goals, and "system parameters." Values used for goals (such as the maximum acceptable compactness of a district) are set or fixed rather than estimated. Goals are subjective in nature, and have no "true" or "universe" values. On the other hand, a "system parameter" (such as the time to conduct routine inspections) is stochastic in nature, but has an underlying "universe" distribution. The value used for this type parameter must be the "best guess" as to the realization of the parameter's value in the future planning period.

Goals

The development of values for each of the goals used in model (P2) is described below.

1. *Inspection Limits:* L_{ik}, U_{ik}. The values used for the upper and lower limit on the frequency of scheduled, routine inspections were based entirely on the judgement of Fire Prevention Bureau management. Values were solicited directly from a group consisting of the Assistant Fire Marshall and two Supervisors. The group reached a consensus on the choice of each value.

It was felt that inspection limits for each occupancy type should be the same regardless of the area of location of the occupant. Therefore, "city-wide" limits were obtained which are independent of census tract considerations.

2. *Maximum Compactness:* C. The value used for the allowable level of the compactness of any district was derived from an analysis of the current set of districts and workloads. Because of the lack of intuitive meaning in the measure of compactness being used, a direct question to solicit a value of the allowable compactness is not practical.

The analysis was based on a computation of the workload required in each census tract resulting from parameter values and inspection rates for 1972 (these rates are the z_k^0 values used in the experiment described in Chapter IV). Census tracts were formed into the 19 districts or territories currently in use. The resulting districts do not coincide exactly with the current territories because of the use of station response territories rather than census tracts in the definition of current territories. However, the center of gravity with respect to workload was estimated (by Bureau Management) for each census tract and used in assigning "split" tracts to a particular station territory or current district. This allowed the district workloads used in this analysis to more accurately parallel the actual workload in each station territory and current district.

After the workload in each current district was determined, the compactness of each district was calculated. The maximum district compactness found was used as the parameter value in the model.

3. *Maximum Workload Deviation*: Δ. The same approach used to set the allowable levelof compactness was used to set the acceptable percentage deviation in the workload of a district from the average. That is, the maximum percentage deviation existing in the current set of districts was used as the parameter value, Δ. The numerical values used for Δ and C are given in Appendix III. Values for L_{ik} and U_{ik} are given in Table 1.

"System Parameters"

The remainder of the parameters used in model (P2) are either random variables, or unknown deterministic factors. For example, the time required to conduct a routine inspection, α_{ik}, is a random variable which may realize different values in repeated observations (i.e., repeated timing for the same i-k). On the other hand, the number of men available for fire prevention duty during the planning period can be considered to have a deterministic value, but is not necessarily known when the model is solved.

The values used in the model for both types of these "system parameters" must be *estimates* of their values to be realized during the future planning period. In the case of the random-variable parameters, an estimate of the mean of the probability distribution of each parameter is used as the parameter value.

Because the planning period used in this application is January, 1974, through December, 1975, estimates of all parameters made now must be *forecasts* of value to occur during this future two-year period. For example, the number of each type structure present in each area during the planning period, N_{ik}, is not known now and must be forecasted. Similarly, all estimates for random-variable parameters must be forecasts of distribution means for the planning period.

Because of a lack of historical data, it was not possible to utilize formal forecasting models and procedures to derive the parameter estimates used in the current solution. Therefore, all numerical estimates used are based on current conditions, and are actual forecasts only to the extent that an implied constant time series forecast model will allow. The continuing data gathering activities of the Fire Department will increase the data base to allow explicit forecasts to be made, as well as enhance the accuracy of all estimates.

The development of an estimate of each "system parameter" will now be described.

1. *Effectiveness Function Slope:* $s_{ik\ell 1}$, $s_{ik\ell 2}$. The development and estimation of these slopes were described in Chapter VI. Their numerical values resulted from an analysis of the data from the first six months of the inspection experiment. The numerical estimates of the slopes are given in Table 4 in Appendix II. (The numerical estimates of

most of the other "system parameters" are presented in Appendix III.)

2. *Number of Structures:* N_{ik}. The current number of type k structures located in area i was used as an estimate of N_{ik}. The Department did not have this information directly, although it did have a list of all buildings which it inspects that was categorized by occupancy type, k (this is file of "route slips"). This was used as the basis of forming a listing of structures by area (census tract) as well as by occupancy type. A computerized version of this list was obtained by having each fire station indicate on the base list which trace each building in their response territory is located. Each station was supplied with a "census map" and the portion of the route-slip list pertaining to its territory. It should be noted that the resulting computerized listing is only as accurate as the original route-slip list, containing thesame duplicatins and omissions.

3. *Adjacent Areas:* $\{A_i\}$. The set of census tracts which is adjacent to each area, $\{A_i\}$, was obtained directly from a census map. Those areas which "touched" or had a common boundary with the tract under consideration were included in $\{A_i\}$. This parameter, unlike the other "system parameters," is completely deterministic and is not subject to forecasting.

4. *Inter-Area Distances:* d_{ij}. In order to calculate the distance between a census tract and a district center, d_{ij}, the center of workload gravity of the tract and the district must be determined. The center of workload of each census tract was estimated by Bureau supervisors who were familiar with the locations of most of the

buildings inspected in each tract. This center of gravity was represented by an arbitrary coordinate system on a census map.

For each district that is defined in the solution process a center of gravity is calculated in the units of the census map coordinates. This is the centroid with respect to all tracts assigned to a district. The two centers of gravity are then used to calculate the euclidean distance between them.

5. *Available Man-Hours:* M. Twenty men were assumed to be available for fire prevention work during the planning period. Each man was assumed to work 40 hours a week for 50 weeks a year. Therefore, it was estimated that 40,000 man-hours would be available. Unexpected sick leave, terminations or additions of men can cause this figure to become inaccurate.

6. *Inspection Time:* α_{ik}. The time required to conduct a routine inspection in an i-k structure (i.e., an occupancy type k structure in area i) was obtained by multiplying an estimate of the average size in square feet of an i-k building by the time to inspect a square foot of that type building. An estimate of the inspection time per square foot was obtained through a special time study of routine inspections. Four inspectors spent a week conducting routine inspections in a sample of structures in each occupancy type category ($k = 1,2,\ldots,34$). The buildings included in the sample were selected by Bureau Management to be representative of other structures in the same occupancy category. The number of buildings sampled from each occupancy category was varied to account for the degree of homogenity within that category. This

number ranged from a minimum of one structure to a maximum of four structures. The time study results were obtained from the information recorded on each inspection, which included (1) the time to complete the inspection, (2) the size of the building inspected, and (3) the number of deficiencies (violations) found.

The average size of an i-k structure was obtained from the list of buildings prepared by station personnel. In constructing the list, station records of past "company inspections" were utilized to determine the size of each building on the list. However, this information was not available for a large portion of the structures listed. For those census tracts for which the number of building sizes given was not sufficient to establish a meaningful average (four was used as an arbitrary cutoff), the average size of the i-k buildings in the time study sample was used.

It should be pointed out that the accuracy of the numerical estimates of α_{ik} is questionable. There are three main reasons for this: lack of a consistent interpretation of the definition of "size" of a building, few buildings sizes are known, and the relatively small time study sample. However, future operations of the Bureau should provide data to improve this accuracy.

7. *Nonscheduled Inspections:* β_{ik}. The percentage of the N_{ik} structures which require nonscheduled, routine inspections was estimated by recording the total number of nonscheduled inspections required over a six-week period. Assuming nonscheduled inspections to occur uniformly over time and over all i-k categories, this six-week total was

extrapolated to obtain a yearly average, then divided uniformly among the i-k categories. Specifically, there were 24 nonscheduled inspections recorded, so that the estimate for each β_{ik} was

$$\hat{\beta}_{ik} = (24)(52)/(6)(118)(34) = .052 \text{ inspections per year for all } i,k,$$

since there are 52 weeks per year, 118 census tracts, and 34 occupancy types.

8. *Fire Report Investigation Time:* γ_i. A special one-week study was conducted to establish an estimate of the time required to conduct a fire report investigation. The average time of actual fire report investigations in all parts of the city during this period was used to estimate γ_i for all areas.

Travel times were also recorded during this one-week study. These times were included in both the fire report investigation time estimate and the routine inspection time estimates.

9. *Number of Fire Report Investigation:* b_i. Historical records of the annual number of fire reports investigated (city-wide) were used to estimate b_i. The total number of investigations conducted in 1972 were averaged over all census tracts to obtain $\hat{b}_i$. Specifically, there were 1568 reports investigated by inspectors in 1972, so that

$$\hat{b}_i = 1568/118 = 13.25 \text{ reports/yr. for all } i.$$

10. *Reinspection Time:* θ_{ik}. The estimate of the time required

to conduct a reinspection of an i-k building, θ_{ik}, was based on the judgement of Bureau Management. They felt that a reliable estimate is that reinspections require 1/4 the time required by a routine inspection. Therefore, the estimate of θ_{ik} used was

$$\hat{\theta}_{ik} = .25\alpha_{ik}.$$

11. *Number of Reinspections:* r_{ik}. Historical data on the total number of reinspections conducted in the city was used to estimate r_{ik}. The total number of reinspections for 1972 was averaged over all i-k categories. There were 2219 reinspections in 1972, so that

$$\hat{r}_{ik} = 2219/(118)(34) = .554 \text{ reinspections/yr. for all } i,k.$$

12. *Indirect Work Parameters.* Estimates for (a) the time required to conduct a nonstructural inspection, ϕ_{ik1}, (b) the time required to answer a complaint, ϕ_{ik2}, and (c) the time required to conduct a new building inspection, ϕ_{i3}, were all obtained directly from Bureau Management. All three parameters were estimated to be the same regardless of the area or occupancy type. The values obtained were

$$\hat{\phi}_{ik1} = 1/2 \text{ hour},$$

$$\hat{\phi}_{ik2} = 3/4 \text{ hour},$$

$$\text{and } \hat{\phi}_{i3} = 1 \text{ hour}, \quad \text{for all } i,k.$$

Estimates for (a) the number of nonstructural inspections, h_{ik1}, (b) the number of annual complaints, h_{ik2}, and (c) the number of new building inspections required, h_{i3}, were all based on 1972 records as reported in the Department's "Annual Report." In all three cases city-wide totals were averaged uniformly over i-k categories. The specific annual estimates obtained were

$$\hat{h}_{ik1} = 136/(118)(34) = .0339,$$

$$\hat{h}_{ik2} = 684/(118)(34) = .171, \quad \text{for all } i,k,$$

$$\text{and } \hat{h}_{i3} = 496/(118) = 4.21, \quad \text{for all } i.$$

The final indirect work parameter, h_4, accounts for the following items: court actions, fire drill supervision, radio and TV talks, lectures, photo work, school drill supervision, fire alarm and exit light checks, and inspector self-education. All the activities are considered to be independent of area and occupancy type. School drills and photo work constitute the bulk of the time consumed by these activities, and are assigned to three specific inspectors as "special duty." These three inspectors spend a considerable amount of their time on this "special duty" and therefore must be assigned small inspection districts.

Estimates for the time required for each component activity of h_4 were obtained directly from Bureau Management. The total hours required for "special duty" for each of the three inspectors were assigned to a census tract in their current inspection district. Specifically, 395 hours were added to the workloads of census tracts 9 and 85 for school activities, and 200 hours added to tract 115 for photo work (the census tract number corresponds to that given in Appendix III). The total of the remaining indirect work was 1092 hours per year. This is the value of h_4.

13. *Potential Seriousness:* v_{ik}. The estimates used for the potential seriousness of an i-k fire were based on subjective assessments made by the group of Bureau Management personnel (Assistant Fire Marshall and two Supervisors). A ratio scale was developed by establishing an arbitrary numerical value for the seriousness of a "neutral" i-k category (a service station outside the fire zone was used). With this value as a base point, the group made judgements as to "how much more potentially serious" is each of the other categories.

The group felt that the value of v_{ik} for each k should differ only with respect to whether or not a structure is located inside or outside the fire zone. Therefore, only two area designations were used in estimating v_{ik}. The value of $\hat{v}_{ik}$ for a specific census tract depends on whether that tract is inside or outside the fire zone. The numerical estimates which were obtained in this procedure are presented in Appendix III.

In obtaining the estimates for all parameters, it was apparent that there were inaccuracies present. Four basic reasons for this which became evident are (1) recording errors on source documents, (2) keypunching errors, (3) interpretation errors by the persons recording source information, and (4) lack of a sufficient amount of data. An example of reasons(3) and (4) was mentioned earlier with respect to inspection times. An example of reason (1) occurred in analyzing the results of the inspection experiment. It is critical in that analysis to match the addresses of fire reports with addresses of inspection reports. Unfortunately, different people were recording these addresses on coding forms and were not entirely consistent in writing the same address in the same format.

Because of these possibilities for error, the data used in the estimation procedures were edited to remove mistakes. However, it should be pointed out that this editing did not remove all errors in the input data.

Several different data sources were used in estimating the model parameters. Some of the estimating procedures required computerized routines. The relationship between these data sources and estimation routines is given in Figure 7.

Model Analysis

The parameter estimates were put into the model and the coded algorithm described in Chapter V was used to obtain a solution. The initial effort of solving the problem resulted in finding no feasible solution. The values of the parameters in the resource usage constraint [constraint (1) in model (P2)] were too large to allow the inspection frequencies, z_{ik}, to be set at their lower limits. This meant that either the lower limit values were unrealistically high, or the resource utilization coefficients (α_{ik}, β_{ik}, etc.) were too high.

A check of the lower limits revealed that, while they do appear

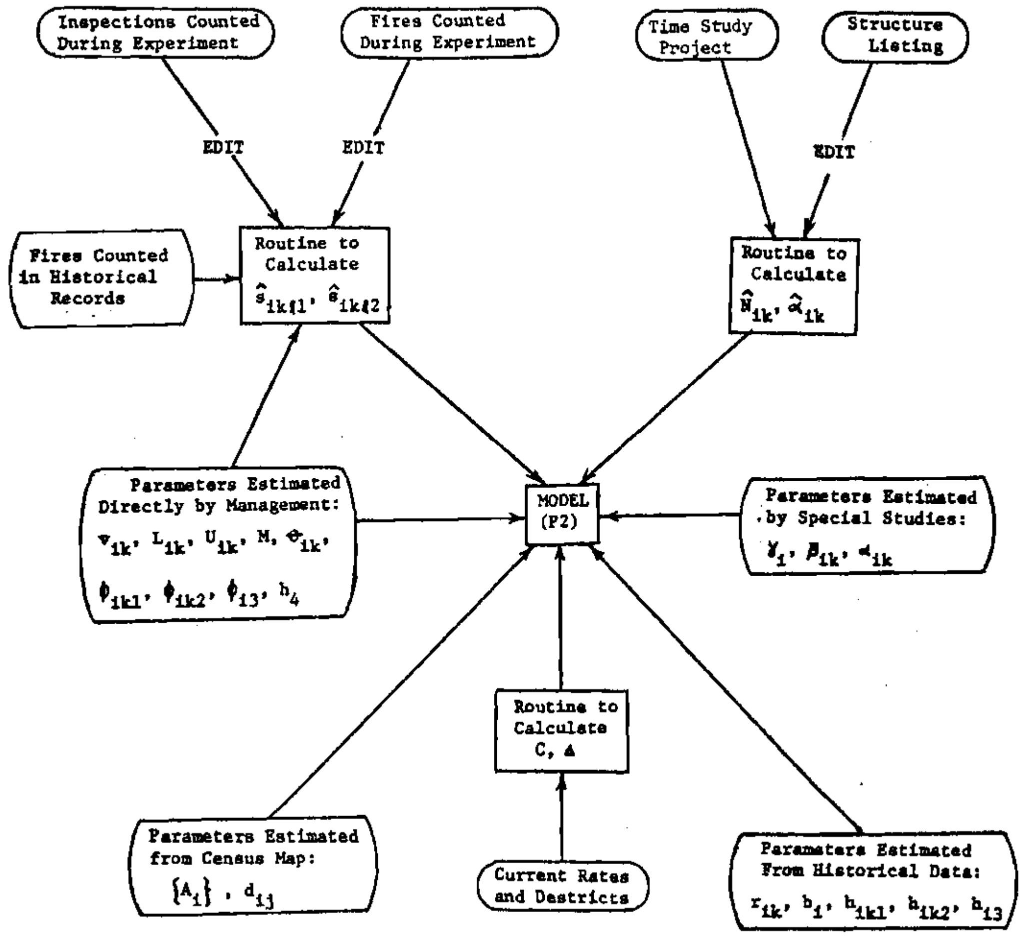


Figure 7. Parameter Estimation Routines and Data Sources

large, they are somewhat parallel to the current inspection rates, z_k^0, and are therefore reasonable. Investigation of the resource utilization coefficients did reveal some unreasonable values. For example, the inspection time for one i-k category was over 50 hours. While this may be reasonable for one or two individual "buildings" in the city (such as the Federal Prison), it is not a realistic estimate for that i-k category.

In order to adjust for such possible inaccuracies, the indirect work parameter h_4 was reevaluated. The value of h_4 was determined which caused the resource usage constraint to be balanced when the current inspection rates are used. A balanced constraint implies that all the available man-hours in 1972 were used by inspection work or other activities. Therefore, the value of h_4 causing this is a reasonable value to use in order that a realistic amount of time be available for inspections.

Using this new value of h_4, the model was resolved. Reasonable inspection frequencies and inspection districts were obtained. Additional solutions were found by varying (1) the set of "guessed" initial district centers, (2) the algorithmic parameters controlling the number of allowable cycles in various parts of the solution procedure, and (3) the magnitude of the penalty attached to noncontiguous areas. Each of the solutions obtained had the same inspection frequencies and had balanced workloads in all districts formed. All solutions were compared on the basis of the districting criteria defined in Chapter V (i.e., workload balance, contiguity, and compactness). The best solution was chosen and constitutes the "final programmed solution."

The inspection rates in the "final programmed solution," are equal to the lower limits on inspection frequency. This resulted from a lack of available man-hours for inspection activity. However, these solution rates are somewhat different than the current rates used by the Bureau. This difference represents a reallocation of inspection efforts over the various occupancy types. The solution rates are given in Table 1, along with upper and lower frequency limits.

The "final programmed solution" had two districts that were not contiguous. A manual reassignment was made of the single census tract causing one of the districts to be noncontiguous. This reassignment resulted in an improvement in all compactness and workload deviation measures. The other noncontiguous district was not adjusted because of the increase in workload inbalance that would be created.

The inspection rates found in the "final programmed solution" and the districts defined by the "final programmed solution" and the one reassignment constitute the final solution of the model. The districts formed are shown in Figure 8. For comparison, the districts currently in use by the Bureau are shown in Figure 9. However, as mentioned earlier, the current districts do not use census tracts as building blocks. Therefore, the districts shown in Figure 9 are not exactly identical to the current districts shown in Figure 2. In forming the current districts using census tracts (Figure 9), the center of workload gravity of each tract was used to decide to which district it should be assigned. Therefore, while the geographic boundaries of the current

Table 1. Solution Inspection Frequencies

		OUTSIDE FIRE ZONE			INSIDE FIRE ZONE		
		Lower Limit	Solution Frequency	Upper Limit	Lower Limit	Solution Frequency	Upper Limit
Occupancy	1	1	1	3	1	1	3
Type	2	1	1	3	1	1	3
Category	3	1	1	3	1	1	3
	4	1	1	3	1	1	3
	5	1/2	1/2	1	1/2	1/2	1
	6	1	1	2	1	1	2
	7	1	1	2	1	1	2
	8	2	2	3	2	2	3
	9	2	2	3	2	2	3
	10	2	2	3	2	2	3
	11	1/2	1/2	1	1/2	1/2	1
	12	2	2	4	2	2	4
	13	2	2	5	2	2	5
	14	1/2	1/2	1	1/2	1/2	1
	15	2	2	4	2	2	4
	16	2	2	4	2	2	4
	17	1	1	2	1	1	2
	18	1	1	2	1	1	2
	19	3	3	6	3	3	6
	20	4	4	6	4	4	6
	21	4	4	6	4	4	6
	22	4	4	6	4	4	6
	23	2	2	4	2	2	3
	24	2	2	3	2	2	3
	25	2	2	3	2	2	3
	26	1	1	2	1	2	2
	27	1	1	2	1	1	2
	28	1	1	2	1	2	2
	29	1/2	1/2	1	1/2	1/2	1
	30	2	2	4	2	2	4
	31	2	2	4	2	2	4
	32	2	2	4	2	2	4
	33	1	1	2	1	2	2
	34	1	1	2	1	1	2

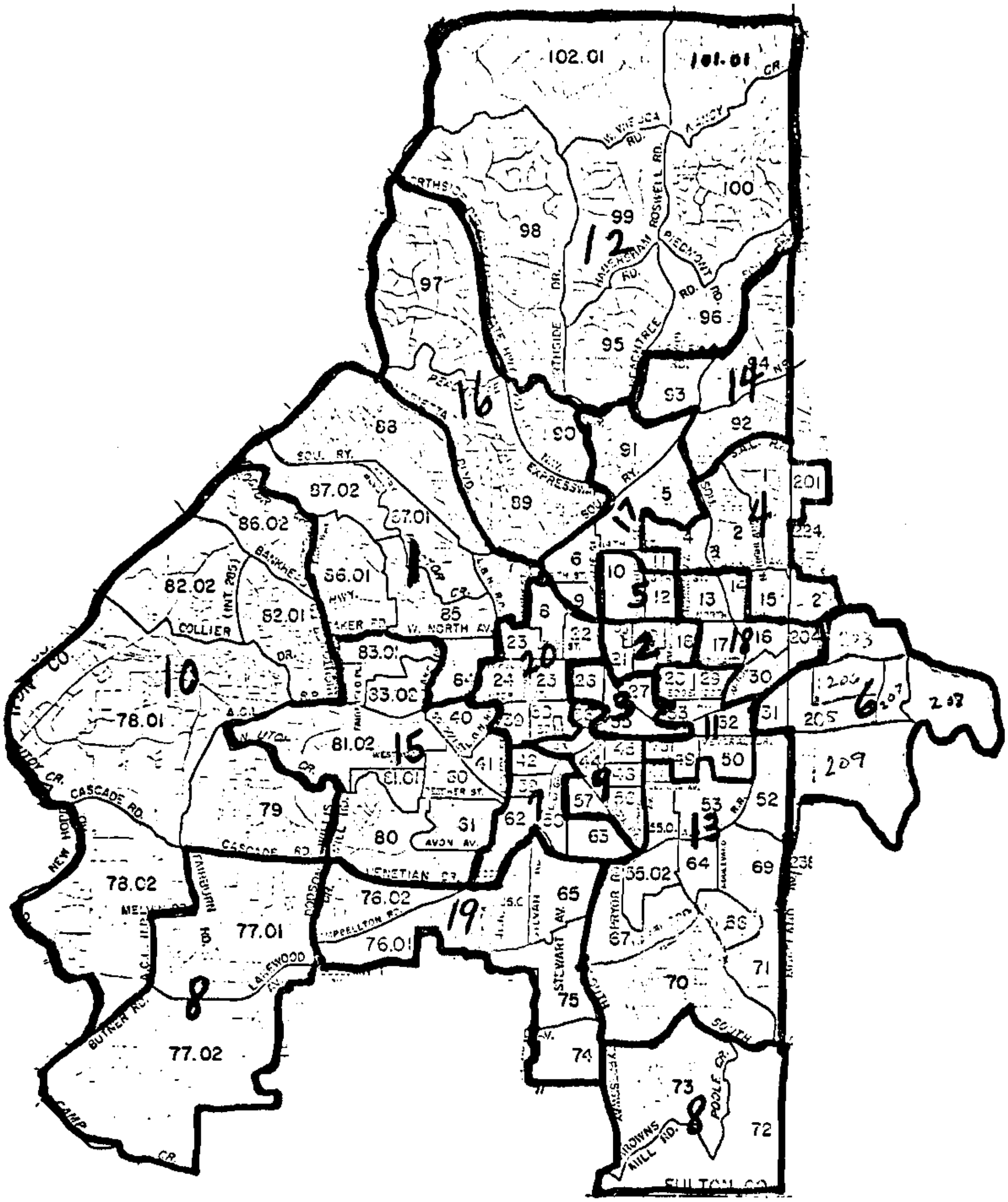


Figure 8. Solution Districts

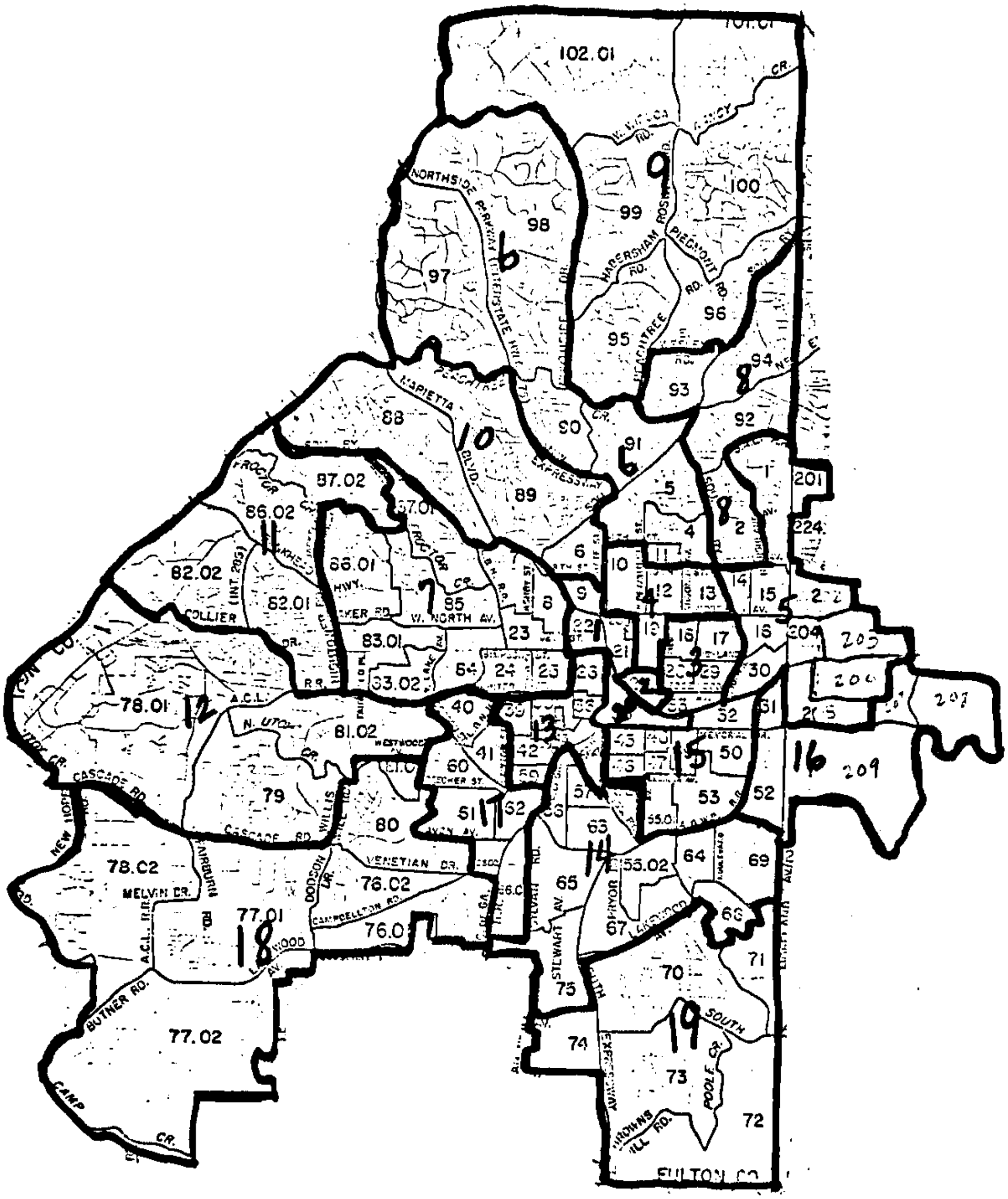


Figure 9. Current Districts

districts based on census tracts do not match those of the current districts based on station response territories, their workload assignments do agree.

CHAPTER VIII

RESULTS AND CONCLUSIONS

This chapter is concerned with the subjective and objective findings of the research, and the conclusions made from examining these results. The conclusions include recommendations for implementation efforts, extensions which can and should be made to the research methodology or the research scope, and the significance of the research. Before describing each of these topics, a summary of the research will be given.

Summary

The purpose of this research was to structure and model several planning problems encountered in conducting fire prevention operations, and to develop solution procedures for these models. An additional goal was to apply the methodology developed for modelling and solving these problems to the operations of the Atlanta Fire Department.

The specific problems addressed are (1) determination of optimum frequencies of scheduled, routine inspections, (2) determination of optimum inspection districts, and (3) determination of an optimal schedule for each routine inspection. All three problems were defined, and the relevant criteria and constraints were described and modelled. A solution procedure for the first two problems was developed and tested on an example problem. The procedure is heuristic, and follows the logic of first finding a superoptimal set of inspection rates, then finding a feasible set of districts for the workload imputed by the superoptimal rates. If a feasible set of districts is not found, the superoptimal rates are modified until feasible districts can be defined.

This "Rate-District" problem was analyzed as it exists in Atlanta. This involved developing estimates for all the parameters in the model. Major effort was devoted to developing estimates of the effectiveness function used in the objective function of the model. An experiment was conducted by the Atlanta Fire Prevention Bureau to determine the effect on the occurrence rate of fires due to changes in building inspection frequencies. The results of the experiment were supplemented with historical data, and with the subjective input of Bureau Management, to derive numerical estimates of the effective functions.

The numerical parameter values were supplied to the model and a set of inspection rates and districts obtained as a solution. This "computer" solution was modified manually by reassigning a noncontiguous census tract to produce a more desirable set of districts.

The parameter estimates and the rate and district solution obtained are intended to be an initial application only. Additional data gathering is being continued by the Fire Department to enhance the accuracy of estimates and, therefore, the usefulness of the problem solution.

Findings

Inspection Effectiveness Experiment

An analysis of the results of the first six months' experimental data revealed that, at the 10 per cent confidence level, there is statistically no improvement in an inspection frequency of once per month, z^2, over a frequency of once per six months, z^1. This was true for each of six occupancy type groups and each of three levels of multiple fires in the same building. The combined z^1 and z^2 samples produced estimates of p(z) which had 90 per cent confidence interval widths that were less than the critical width of .0235 for 12 of the 18 j-ℓ sample categories.

A comparison of current inspection policy with one inspection per six months revealed 14 categories in which there was a statistical improvement of the six month inspection rate. These k-ℓ categories are: 4-1, 5-1, 7-1, 18-1, 22-1, 23-1, 25-1, 26-1, 28-1, 29-1, 30-1, 31-1, 32-1, and 33-1. Each of the 14 "k" values is either in the experimental occupancy type group j = 3 or in the group j = 6.

An analysis of the historical data used to estimate $p(z_k^0)$ also revealed no evidence of seasonal influence in the occurrence rate of nondwelling structural fires (see Table 2 in Appendix II). There was a feeling in the Bureau before this finding that this influence was very significant.

The results of the analysis of this first six months' experimental data does not indicate any conclusive findings. Additional experimentation must be performed before reliable estimates of p(z) can be obtained. There are too many extraneous and random factors present to allow this initial experimental result to be conclusive. In addition, the question of "how much data is enough" needs to be more concretely answered.

Inspection Rates and Districts

The specific inspection rates and districts found as the solution for the "Rate-District" problem as it applies to Atlanta are given in Table 1 and Figure 8, respectively. The districts found appear to be superior to the current set of districts used by the Bureau (see Figure 8). A comparison of the two sets of districts revealed the following values:

	Current Districts	Solution Districts
Sum of Squares of Deviation of District Workloads from Average	14,772,974	469,062
Maximum Percentage Absolute Workload Deviation for a District	69%	14%
Maximum District Workload	1563	326
Average District Compactness	74,783	87,239

The above numerical comparison is based on workloads resulting from use of the proposed inspection rates rather than current inspection rates. However, a similar comparison was found when current rates were used.

It should be observed that while the workload deviation measures are all in favor of the solution districts, the average compactness of the current districts is smaller than that of the solution districts. This occurs primarily because two of the current districts are composed of only one census tract, implying a zero compactness measure.

A final result or observation concerning the analysis of the "Rate-District" problem is that estimates of model parameters are not entirely accurate. Most of the inaccuracies appear to be present in estimates of α_{ik}. Other parameter values appear to be reasonable.

Recommendations

Because of the possible inaccurate estimates, it is recommended that additional data be gathered and used to obtain more reliable parameter estimates. Before any decision is made on either inspection frequencies or inspection districts, the more reliable estimates should be used in the solution process. However, it is recommended that Bureau Management review the significance of the proposed rates and districts, and begin planning implementation efforts. Problems which will arise in changing from current district assignments to census-tract-based assignments should be investigated. For example, the change over will involve disruptions in record keeping, and the associations between inspectors and occupants. Also, because new districts will cause assignments to be overlapping in most station response territories, disruptions will occur in communication between station personnel and inspectors.

The Bureau should continue to pursue its efforts to develop and implement a complete "computerized inspection program." Data gathering efforts should be continued and expanded to include objective information on such parameters as indirect work components. The potential advantages to the Bureau of using of a computerized inspection program (including optimal setting of rates, districts, and schedules) includes the following.

1. Use of more balanced districts will provide more equality in the coverage or inspection service given to the same category of occupants, regardless of to whose district they are assigned.

2. Use of more balanced districts will provide more equality in the total inspection and noninspection work assigned to all inspectors.

3. Man-hours available for inspections can be more effectively used by inspecting those categories having more value to the Bureau.

4. More control over the activities of the inspectors will be possible through "management report" type feedback. This might lead to more efficient use of available man-hours, and therefore more inspections might be made.

5. There is a potential for a reduction in the number of supervisors required in management of inspector activities.

6. Easier and more effective updating of district assignments can be made as personnel changes are made.

7. Meaningful evaluation of the impact on prevention operations can be made as potential personnel changes are reviewed. This factor should allow budget recommendations to be justified.

Extensions

The following extensions or modifications to the methodology used in this research would enhance its attractiveness.

1. Additional heurictrics should be included in the solution

procedure to improve the workload balance of each district. Also, this type heurictric should be applied to a district even though its workload is within acceptable limits.

2. Improved procedures need to be included in the solution. Method to ensure that contiguous districts are defined.

3. An exact solution algorithm for the "Rate-District" problem needs to be developed.

4. Explicit forecasting procedures need to be developed and used in estimating all parameter values.

The following are extensions of the scope of the research, or additional research areas which need to be investigated.

1. The methodology and models developed in this research may be applicable to other problem areas. For example, the determination of sales calls and salesman districts in private sector organizations is almost identical to the "Rate-District" problem. In addition, there are several other application areas in the public sector, such as police patrolling frequencies and beat determination, building inspections by the Building Department, and generally any agency which provides inspection service.

2. Additional organizational schemes should be modelled, such as the use of "specialized" inspectors responsible for only specific occupancy types, and "company" inspections conducted by fire station personnel.

3. A solution procedure needs to be developed for the scheduling problem.

4. The use of the developed models to (a) evaluate budget requests, and (b) allocate fire prevention manpower between inspection and public education activity needs to be investigated.

5. The effect of inspection frequency level on the seriousness of fires needs to be established.

6. The desirability of alternative schedules of inspections with respect to time between inspections (in the same building) and location of "simultaneous" inspections needs to be established.

In concluding this report, it might be useful to emphasize the significance of this research. A contribution to the existing "body of knowledge" resulted from five research activities. They are (1) structuring an existing, real-world problem which had not been previously researched, (2) developing a mathematical model for this problem, (3) developing a solution procedure for the problem, (4) applying the model and solution method to an operating fire department (including deriving all required numerical estimates and obtaining a problem solution), and (5) developing the methodology required to estimate the inspection effectiveness function, and deriving a numerical estimate of this function for an operating fire department.

APPENDIX I

INSPECTION EFFECTIVENESS EXPERIMENT

The design of the experiment used to obtain objective estimates of the effect of inspection rates on the occurrence of fires was described in Chapter IV. Some additional information relevant to the experiment is given below.

Occupancy Type Categories

The specific occupancy types included in each of the six experimental categories were as follows:

Category $j = 1$: Occupancy types 27, 39, 40, 41, 42, 43, 44.

Category $j = 2$: 35, 36, 37, 45, 46, 47.

Category $j = 3$: 1, 5, 38, 48, 49, 50, 52, 53, 54, 59.

Category $j = 4$: 16, 22, 26, 30, 31, 32, 56, 57, 58, 61, 62.

Category $j = 5$: 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 23, 24, 25, 28, 29, 33, 60.

Category $j = 6$: 2, 3, 18, 21, 34, 55, 63, 64, 65, 66.

Sample Sizes

The size of the samples used to estimate $p(z^0)$ were the total number of structures of each occupancy type category (j) in the city in 1971. The sizes of the samples used to estimate $p(z^1)$ and $p(z^2)$ were found by counting the number of relevant structures as they were processed in the computer analysis. These sample sizes deviated from those initially set up because of key-punch error, lost reports, etc. The samples sizes used are given below.

	z^0	z^1	z^2
j = 1	1568	168	262
2	1075	128	208
3	5948	783	1206
4	1168	62	116
5	1678	34	59
6	2108	159	262

Report Forms

The coding forms used to collect data on each inspection conducted during the experiment and on each structural fire occurring are given in Figure 9 and Figure 10, respectively. The form shown in Figure 10 follows the format given in Figure 11. The basic source of most of the information in the form in Figure 10 came from a special supplemental fire report compiled by the Battalion Chiefs at each fire. This form is shown in Figure 12.

FIRE STUDY REPORT FORM 2

(Inspections)

1. Address
St.No. St. Name St. Type St. Dir

2. Date of Inspection Month Day Year

3. Station Area Number

4. Type structure (No. in list in Annual Report)

5. Construction Material (Nonresistent = 0
Resistent = 1)

6. Complete Sprinkler System (No = 0, Yes = 1)

7. A. Number of structural violations found

B. Number of "housekeeping" violations found

8. Age of structure (years)

9. Experimental Inspection Rate
(Number per 6 months: 1, 6)

10. Number of stories

11. Size of structure (square feet)

12. Date of last inspection Month Day Year

13. Signed: ______________________________

Figure 10. Coding Form for Inspections

FIRE STUDY REPORT FORM 3

NOTE: (1) This form is to be filled out for all structural fires and calls involving injuries
(2) "ø" represents "zero"

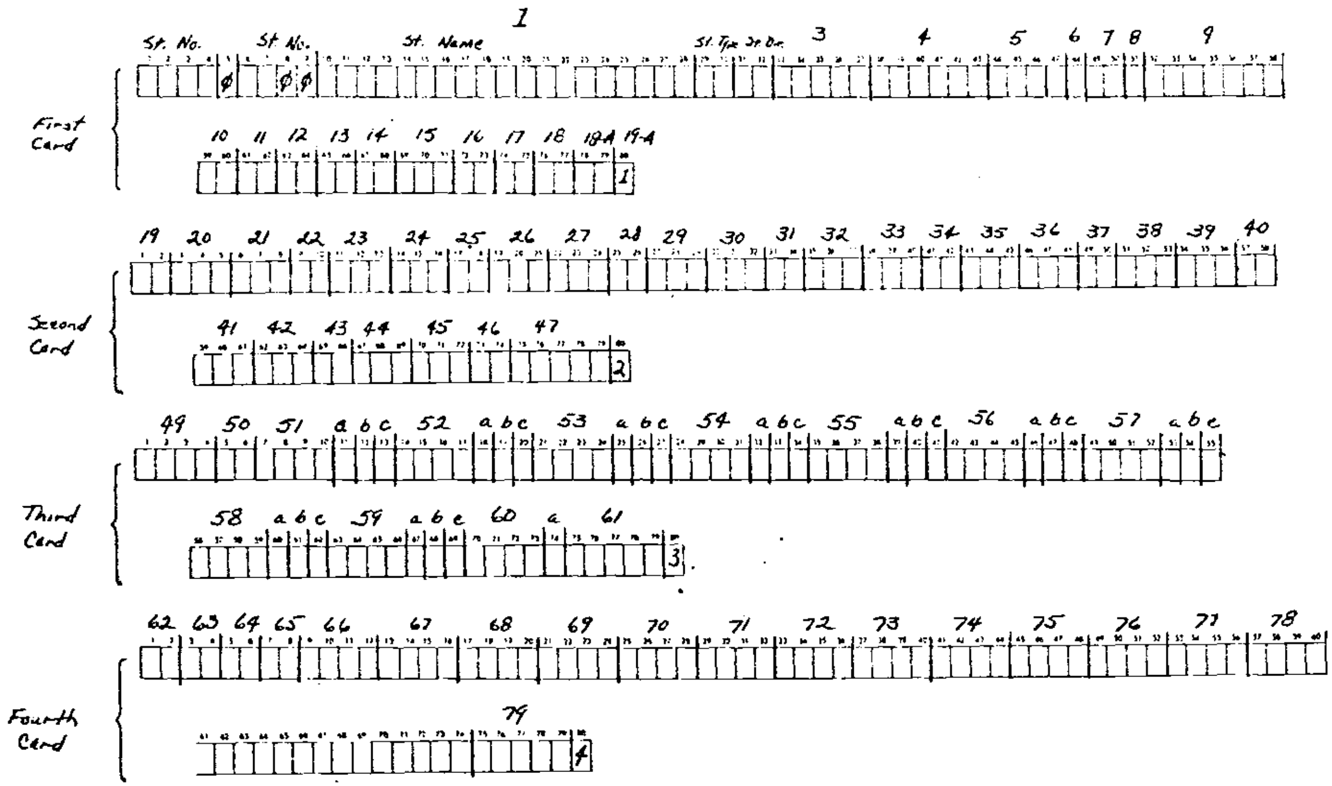


Figure 11. Coding Form for Fires

FIRE REPORT CODING FORM

ORDER	SOURCE	DATA	FORMAT
1	FR	Address of Fire	
3	FR	Fire Number	
4	FR	Date of Fire	2 digits for month, day & year (i.e., May 1, 1973 to be written 05 01 73)
5	FR	Time of Alarm Receipt	"2400" Hour Time
6	FR	Type Construction	1 - 7
7	FR	Occupancy Type	15-68
8	FR	Occupancy Type Category	1 - 6
9	FR	Total Dollar Loss	Round off to nearest $
10	FR	Cause of Fire	If fire is under investigation, leave Order #10 blank until a final cause of fire is determined by Investigator
11	FR	Number Extinguishers Used	
12	FR	Number Booster Hoses Used	Express as "075"
13	FR	Number 1½ Inch Hoses Used	Express as "150"
14	FR	Number 2½ Inch Hoses Used	Express as "250"
15	Form 1.2	Percentage of Initial Involvement	
16	Form 1.3	Number of Men on Scene from Engine Co.s	
17	Form 1.3	Number of Men on Scene from Ladder Co.s	
18	Form 1.4	Number of Alarms Given	
18-A		BLANK	
19-A		~~BLANK~~ 201	

FIRST CARD 1 - 19A

Figure 12. Format for Coding Form for Fires

ORDER	SOURCE	DATA	FORMAT
19	Form 1.4	Alarm Number for First Hose	
20	Form 1.4	Size Hose Used	
21	Form 1.4	Number of Minutes After Arrival Until First Hose Used	
22	Form 1.4	Alarm Number for Second Hose	
23	Form 1.4	Size Hose Used	
24	Form 1.4	Number of Minutes After Arrival Until Second Hose Used	
25	Form 1.4	Alarm Number for Third Hose	
26	Form 1.4	Size Hose Used	
27	Form 1.4	Number of Minutes After Arrival Until Third Hose Used	
28	Form 1.4	Alarm Number for Fourth Hose	
29	Form 1.4	Size Hose Used	
30	Form 1.4	Number of Minutes After Arrival Until Fourth Hose Used	
31	Form 1.4	Alarm Number for Fifth Hose	
32	Form 1.4	Size Hose Used	
33	Form 1.4	Number of Minutes After Arrival Until Fifth Hose Used	
34	Form 1.4	Alarm Number for Sixth Hose	
35	Form 1.4	Size Hose Used	
36	Form 1.4	Number of Minutes After Arrival Until Sixty Hose Used	
37	Form 1.4	Alarm Number for Seventh Hose	
38	Form 1.4	Size Hose Used	
39	Form 1.4	Number of Minutes After Arrival Until Seventh Hose Used	
40	Form 1.4	Alarm Number for Eighth Hose	
41	Form 1.4	Size Hose Used	
42	Form 1.4	Number of Minutes After Arrival Until Eighth Hose Used	
43	Form 1.4	Alarm Number for Ninth Hose	
44	Form 1.4	Size Hose Used	
45	Form 1.4	Number of Minutes After Arrival Until Ninth Hose Used	
46	Form 1.4	Alarm Number for Tenth Hose	
NO 47	Form 1.4	Size Hose Used	
~~48~~	Form 1.4	Number of Minutes After Arrival Until Tenth Hose Used	

46 Blank

47 Fire No.

SECOND CARD 19 - 48

Figure 12. Continued

ORDER	SOURCE	DATA	FORMAT
49	Form 1.5	Time of Fire Ignition	"2400" Hour Time
50	Form 1.5	Number of Civilian Injuries	
51	Form 1.5	Time of First Injury	"2400" Hour Time
51-A	FR	Extent of First Injury	*
51-B	Form 1.6	Extent of First Injury if Arrival Time Decreased	
51-C	Form 1.7	Extent of First Injury if Arrival Time Delayed	
52	Form 1.5	Time of Second Injury	"2400" Hour Time
52-A	FR	Extent of Second Injury	*
52-B	Form 1.6	Extent of Second Injury if Arrival Time Decreased	
52-C	Form 1.7	Extent of Second Injury if Arrival Time Delayed	
53	Form 1.5	Time of Third Injury	"2400" Hour Time
53-A	FR	Extent of Third Injury	*
53-B	Form 1.6	Extent of Third Injury if Arrival Time Decreased	
53-C	Form 1.7	Extent of Third Injury if Arrival Time Delayed	
54	Form 1.5	Time of Fourth Injury	"2400" Hour Time
54-A	FR	Extent of Fourth Injury	*
54-B	Form 1.6	Extent of Fourth Injury if Arrival Time Decreased	
54-C	Form 1.7	Extent of Fourth Injury if Arrival Time Delayed	
55	Form 1.5	Time of Fifth Injury	"2400" Hour Time
55-A	FR	Extent of Fifth Injury	*
55-B	Form 1.6	Extent of Fifth Injury if Arrival Time Decreased	
55-C	Form 1.7	Extent of Fifth Injury if Arrival Time Delayed	
56	Form 1.5	Time of Sixth Injury	"2400" Hour Time
56-A	FR	Extent of Sixth Injury	*
56-B	Form 1.6	Extent of Sixth Injury if Arrival Time Decreased	
56-C	Form 1.7	Extent of Sixth Injury if Arrival Time Delayed	
57	Form 1.5	Time of Seventh Injury	"2400" Hour Time
57-A	FR	Extent of Seventh Injury	*
57-B	Form 1.6	Extent of Seventh Injury if Arrival Time Decreased	
57-C	Form 1.7	Extent of Seventh Injury if Arrival Time Delayed	
58	Form 1.5	Time of Eighth Injury	"2400" Hour Time
58-A	FR	Extent of Eighth Injury	*
58-B	Form 1.6	Extent of Eighth Injury if Arrival Time Decreased	
58-C	Form 1.7	Extent of Eighth Injury if Arrival Time Delayed	

Figure 12. Continued

-4-

ORDER	SOURCE	DATA	FORMAT
59	Form 1.5	Time of Ninth Injury	"2400" Hour Time
59-A	FR	Extent of Ninth Injury	*
59-B	Form 1.6	Extent of Ninth Injury if Arrival Time Decreased	
59-C	Form 1.7	Extent of Ninth Injury if Arrival Time Delayed	
60	Form 1.5	Time of Tenth Injury	"2400" Hour Time
60-A	FR	Extent of Tenth Injury	*
60-B	Form 1.6	Extent of Tenth Injury if Arrival Time Decreased	
60-C	Form 1.7	Extent of Tenth Injury if Arrival Time Delayed	
61		BLANK	

No

60 Blank
60-A Blank
61 Fire No.

* EXTENT OF INJURY TO BE EXPRESSED AS FOLLOWS:

1 = Minor

2 = Treated by Physician & Released

3 = Hospitalized

4 = Death

THIRD CARD 49 - 61

Figure 12. Continued

-5-

ORDER	SOURCE	DATA	FORMAT
62	Form 1.8	Number of Civilians Trapped & Rescued	
63	Form 1.9	Number of Civilians Self-Evacuated	
64	Form 1.10	Number of Civilians Evacuated from All Structures As Precaution	
65	Form 1.7	Total Extent of Injuries Caused to Civilians Evacuated or Rescued if Arrival Time Delayed	ADD all increases together for ALL evacuees
66	Dispatch	Arrival Time of 1st Alarm Force	"2400" Hour Time
67	Dispatch	Arrival Time of 2nd Alarm Force	"2400" Hour Time
68	Dispatch	Arrival Time of 3rd Alarm Force	"2400" Hour Time
69	Dispatch	Arrival Time of 4th Alarm Force	"2400" Hour Time
70	Dispatch	Arrival Time of 5th Alarm Force	"2400" Hour Time
71	Dispatch	Arrival Time of 6th Alarm Force	"2400" Hour Time
72	Dispatch	Arrival Time of 7th Alarm Force	"2400" Hour Time
73	Dispatch	Arrival Time of 8th Alarm Force	"2400" Hour Time
74	Dispatch	Arrival Time of 9th Alarm Force	"2400" Hour Time
75	Dispatch	Arrival Time of 10th Alarm Force	"2400" Hour Time
76	Dispatch	Arrival Time of 11th Alarm Force	"2400" Hour Time
77	Dispatch	Arrival Time of 12th Alarm Force	"2400" Hour Time
78	Dispatch	Arrival Time of 13th Alarm Force	"2400" Hour Time
79	Fire No.		

FOURTH CARD 62 - 78

Figure 12. Continued

FIRE STUDY REPORT FORM 1

1. Location ______________________ Fire No. ______

2. Estimated percentage of structure initially involved in fire (%) [ ][ ][ ]

3. Number of men on scene from engine vehicle [ ][ ]

4. Number of men on scene from ladder vehicle [ ][ ]

5. Age of structure involved (years) [ ][ ]

6. For each hose actually used (to get water on the fire) give the engine company operating it, its size and the time at which it is first used to put water on the fire.

(A) FIRST ALARM FORCE

	Engine Company #	Time	Size Hose Used
1.			
2.			
3.			
4.			

(B) SECOND ALARM FORCE

	Engine Company #	Time	Size Hose Used
1.			
2.			
3.			
4.			

(C) THIRD ALARM FORCE

	Engine Company #	Time	Size Hose Used
1.			
2.			
3.			
4.			

(D) FOURTH ALARM FORCE

	Engine Company #	Time	Size Hose Used
1.			
2.			
3.			
4.			

7. Time of fire ignition ________

Figure 13. Supplemental Fire Report

8. Civilian Injuries:

   (A) Time of Occurrence 1._____ 2._____ 3._____ 4._____ 5._____

   (B) Extent* 1._____ 2._____ 3._____ 4._____ 5._____

   *Extent: Indicate minor injuries (treated on scene) with the number "1." Indicate injuries other than minor with the number "2." Follow up on injuries other than minor will be the responsibility of the Arson Squad.

9. To what extent (extent = minor, treated by a physician and released, hospitalized and death) would each of the injuries have been lessened by a decrease in arrival time of the first alarm units?

   ____

   ____

   ____

10. To what extent would each of the injuries have been increased and each of the evacuees been injured by a delay in arrival time of the first alarm units? __________________________

   ____

   ____

11. Number of civilians trapped in the immediate structure and rescued by department personnel ☐☐

12. Number of civilians temporarily trapped in the immediate structure and self-evacuated ☐☐

13. Number of civilians evacuated from immediate and surrounding structures as a precautionary measure ☐☐

SIGNED: ____________________________

Figure 13. Continued

APPENDIX II

EXPERIMENT RESULTS

The numerical results of the experiment described in Chapter IV are presented in this appendix. The number of fires counted each $j = 1,2,\ldots,6$ and $\ell = 1,2,3$ category from historical records for the z^0 samples is given in Table 2. Results are given for both six month intervals: April-September ($t = 1$), and October-March ($t = 2$).

The number of fires counted in each j-ℓ category in the experiment is given in Table 3 for both z^1 and z^2 samples. These values were used to test the hypothesis that $p(z^1) = p(z^2)$. The hypothesis was accepted for all j-ℓ categories. After combining the sample results for z^1 and z^2 samples, the composite sample result for each j-ℓ category was tested for adequacy of the width of the associated confidence interval. A critical width value of .0235 was used. This is the maximum width of any j-ℓ-z^0 confidence interval found in analyzing historical data. The widths for each j-ℓ category are given below.

	$j = 1$	2	3	4	5	6
$\ell = 1$	.009	.027	.006	.038	.063	.010
2	.009	.017	.004	.025	.048	.006
3	.005	.007	.004	.015	.030	.006

Table 2. Fire Count from Historical Data

	t = 1			t = 2		
	ℓ = 1	2	3	ℓ = 1	2	3
j = 1	21.5	1.0	.5	24.5	1.0	.5
2	55.5	2.0	4.0	51.0	4.0	2.0
3	55.5	2.0	.5	51.5	2.5	.5
4	23.0	1.0	0.0	20.0	2.0	0.0
5	11.5	.5	0.0	9.0	1.0	0.0
6	59.0	.5	0.0	52.5	1.0	.5

(These values are six month totals averaged of the two year period 1970-1972.)

Table 3. Fire Count from Experimental Data

	z^1			z^2		
	ℓ = 1	2	3	ℓ = 1	2	3
j = 1	1	0	0	0	1	0
2	1	1	0	5	1	0
3	6	0	0	5	2	0
4	0	0	0	3	1	0
5	0	1	0	2	0	0
6	0	0	0	1	0	0

Effectiveness Functions

The objective estimates of $p_{j\ell t}(z)$ obtained from the analysis of the experiment results, and the subjective estimates of $p_{k\ell t}(z)$ provided by management, were used to provide a composite probability estimate for z^0, z^1, and z^2. These composite probabilities were used to form an estimate of the inspection effectiveness function according to procedures described in Chapter IV. The slopes the piece-wise linear functions which were formed are given in Table 4. Note that the estimated slopes are independent of area (i) considerations.

Table 4. Effectiveness Function Slopes
($\times 10^{-7}$)

	$\ell = 1$		$\ell = 2$		$\ell = 3$	
	$s_{k\ell 1}$	$s_{k\ell 2}$	$s_{k\ell 1}$	$s_{k\ell 2}$	$s_{k\ell 1}$	$s_{k\ell 2}$
k = 1	167	167	23	23	3	3
2	33000	13200	2900	725	0	0
3	104	104	157	157	0	0
4	11000	33000	51	8	60	60
5	600	0	17	0	27	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	2121	0	240	0	0	0
9	0	0	0	0	0	0
10	622	0	40	0	36	0
11	209	0	63	0	0	0
12	0	0	0	0	0	0
13	5444	3889	260	160	0	0
14	78	0	6	0	0	0
15	0	0	0	0	0	0
16	3400	1889	360	160	0	0
17	2000	0	504	0	0	0
18	21667	0	201	0	150	0
19	1911	1911	93	93	47	47
20	2425	2425	323	323	175	175
21	3400	3400	966	966	732	732
22	62	62	6	6	0	0
23	1720	0	18	0	32	0
24	14880	0	845	0	732	0
25	300	0	33	0	4	0
26	1939	0	40	0	5	0
27	1560	0	228	0	173	0
28	267	0	50	0	7	0
29	8895	0	56	0	7	0
30	200	200	23	23	3	3
31	100	100	12	12	1	1
32	4320	4320	8	8	6	6
33	333	0	50	0	7	0
34	0	0	0	0	0	0

APPENDIX III

MODEL APPLICATION

Parameter Estimates

The numerical values of some of the parameters of the model described in Chapter V which were used in the Atlanta application are given in Chapter VII. The values of α_{ik} and N_{ik} are too many to give here. Other important parameter values are given in Table 5, along with the specific occupancy type which constitute each of the $k = 1,2,\ldots,34$ occupancy type categories used in the model (and the experimental category $j = 1,2,\ldots,6$ to which each k belongs). These values are for inspection times (per square foot) and the potential seriousness of a fire, v_{ik} (estimated only for location inside the fire zone and for those outside the zone). The current inspection rate, z_k^0, is also given for each occupancy category.

The values of the parameters given in Chapter VII are repeated below.

$$\hat{\beta}_{ik} = .052, \quad \hat{\theta}_{ik} = .25\alpha_{ik}, \quad \hat{r}_{ik} = .554,$$

$$\hat{\gamma}_i = .6331, \quad \hat{b}_i = 13.25, \quad \hat{\phi}_{ik1} = .5, \quad \hat{\phi}_{ik2} = .75$$

$$\hat{\phi}_{i3} = 1.0, \quad \hat{h}_{ik1} = .0339, \quad \hat{h}_{ik2} = .171, \quad \hat{h}_{i3} = 4.21,$$

$$\hat{h}_4 = 1982.5, \quad M = 40000, \quad \text{and travel time} = .321.$$

The remaining parameter values used in the model are for the goals Δ and C. These values are based on the maximum district compactness and workload deviation of the current set of districts. The census tracts used to define these districts are given in Table 6. The resulting value of Δ is .699 and of C is 258,517. (The numbering system used for census tracts in Table 7 is different than the actual census tract numbers in the map in Figure 7; the correspondence is shown in Table 8.)

Solution

The inspection frequencies found in the solution procedure were given in Chapter VII. A map of the districts defined was given in Figure 7. The specific census tracts making up those districts is given in Table 7. (Again, the census tract numbering is slightly different than the original census may numbering; see Table 8.)

Table 5. Model Parameters

	Occupancy Types	Experimental Category	Inspection Time (min/ft^2)	Potential Seriousness In Zone	Potential Seriousness Out of Zone	z_k^0
k=1	1	3	.0190	25	15	2
2	22,56,58,61,62	4	.008130	25	15	1
3	4,6,7,8,11	5	.008704	25	15	3
4	66	6	.005868	25	15	1
5	2,3	6	.013686	4	1	1/2
6	9	5	.007244	25	15	3
7	34	6	.007244	25	15	3
8	10,20,68	5	.01484	40	20	3
9	16	4	.002519	40	20	3
10	21,63	6	.00380	40	20	2
11	67	5	.010530	2	1	3
12	12,13	5	.006250	80	60	3
13	14,15	5	.002885	50	25	3
14	69	5	.009167	10	5	1/2
15	32	4	.018262	60	40	3
16	17,19,23,33,60	5	.003397	60	40	3
17	31,57	4	.011926	25	15	2
18	18,64,65	6	.008784	25	15	1
19	27	1	.003079	80	60	3
20	39,40,43	1	.002860	80	60	4
21	35,36,37,45	2	.00520	80	60	2
22	38	3	.001985	80	60	4
23	42,44	1	.009110	30	15	1
24	51	2	.003360	30	15	1
25	5,59	3	.003116	30	15	1
26	41	3	.001632	40	20	3/4
27	46	2	.0150	40	20	1
28	48	3	.0180	40	20	1
29	50	3	.008036	20	10	1/2
30	52,53	3	.00683	70	60	1
31	54	3	.002520	60	50	1
32	55	6	.004660	60	50	1
33	49	3	.000604	40	20	1
34	47	7	.002720	40	20	1/2

Table 6. Current District Assignments

District	Census Tracts Assigned
1	8, 19, 20, 21
2	26
3	16, 17, 27, 28, 32
4	9, 11, 12, 18
5	1, 13, 14, 15, 29, 107, 108, 109, 110, 111, 112
6	3, 4, 10, 96, 97, 103, 104
7	6, 7, 22, 23, 24, 88
8	2, 98, 99, 100
9	10, 102, 105, 106, 117, 118
10	5, 94, 95
11	78, 80, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93
12	33
13	25, 34, 35, 36, 37, 40, 41, 42, 56
14	52, 53, 54, 55, 60, 61, 62, 65, 66, 67, 73
15	31, 43, 44, 45, 46, 47, 48, 50, 51
16	30, 49, 113, 114, 115
17	38, 39, 57, 58, 59, 63, 64
18	74, 75, 76, 77, 79, 81, 82
19	68, 69, 70, 71, 72, 116

Table 7. Proposed District Assignments

District	Census Tracts Assigned
1	6, 88, 89, 90, 92, 93, 94
2	17, 18, 19, 20
3	25, 26, 33
4	1, 2, 3, 14, 107, 108
5*	9, 11
6*	3, 109, 111, 112, 113, 114, 115
7	4, 55, 56, 59, 60, 64
8	7, 71, 76, 77, 116
9	34, 41, 42, 43, 44, 53, 54
10*	78, 79, 80, 84, 85, 91
11	27, 28, 31, 32, 46, 47, 48
12	101, 102, 104, 105, 106, 117, 118
13	45, 49, 50, 51, 52, 61, 65, 66, 67, 68, 69
14	98, 99, 100
15	38, 39, 57, 58, 81, 82, 83, 86, 87
16	95, 96, 103
17	4, 5, 8, 10, 97
18	12, 13, 15, 16, 29, 110
19	62, 63, 72, 73, 74, 75
20	7, 21, 22, 23, 24, 35, 36, 37

*Special duty in addition.

Table 8. Census Tract Numbering System

Census Map Number	New Number	Census Map Number	New Number	Census Map Number	New Number
1	1	43	41	80	81
2	2	44	42	81.01	82
4	3	45	43	81.02	83
5	4	46	44	82.01	84
6	5	47	45	82.02	85
7	6	48	46	83.01	86
8	7	49	47	83.02	87
9	8	50	48	84	88
10	9	52	49	85	89
11	10	53	50	86.01	90
12	11	55.01	51	86.02	91
13	12	55.02	52	87.01	92
14	13	56	53	87.02	93
15	14	57	54	88	94
16	15	58	55	89	95
17	16	59	56	90	96
18	17	60	57	91	97
19	18	61	58	92	98
20	19	62	59	93	99
21	20	63	60	94	100
22	21	64	61	95	101
23	22	65	62	96	102
24	23	66.01	63	97	103
25	24	66.02	64	98	104
26	25	67	65	99	105
27	26	68	66	100	106
28	27	69	67	201	107
29	28	70	68	202	108
30	29	71	69	203	109
31	30	72	70	204	110
32	31	73	71	205	111
33	32	74	72	206	112
35	33	75	73	207	113
36	34	76.01	74	208	114
37	35	76.02	75	209	115
38	36	77.01	76	Airport	116
39	37	77.02	77	102.01	117
40	38	78.01	78	101.01	118
41	39	78.02	79		
42	40	79	80		

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